How to Commit (If You Must): Commitment Contracts and the Dual-Self Model

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November 8, 2012

Abstract

This paper studies how dual-self (Fudenberg and Levine (2006)) decision-makers can use commitment technologies to combat temptation and implement long-run optimal actions. I consider two types of such technologies: carrot contracts (rewards for ‘good’ behavior financed by borrowing from future consumption) and stick contracts (self imposed fines for ‘bad’ behavior). Both types of contracts can simulate binding commitment when it is not available and thus offer such contracts make DMs better off. I show that the exact properties of optimal carrots and sticks depend crucially on the short-run discount rate and the time it takes for the contract to ‘kick in.’ Finally, I compare the welfare implications of these contracts and show that dual-self decision-makers strictly prefer to use carrots instead of either sticks or binding commitments. This is for several reasons: sticks are highly vulnerable to trembles (while carrots are not), sticks and binding commitments create a temptation to cancel them (while carrots do not), and finally carrots allow easy tradeoffs between commitment and flexibility (while sticks and binding commitments do not).

*Dept. of Economics, Harvard University. Contact: apeysakh@fas.harvard.edu. I thank Dan Burghart, Adam Brandenburger, Paul Glimcher, Drew Fudenberg, Oliver Hart, Yuichiro Kamada, David Laibson, Aurélie Ouss, David Rand, Al Roth, Tomasz Strazlecki, Ryan Webb, and the participants of the Harvard Economics Theory Lunch for incredibly helpful advice and comments on this project. I thank anonymous referees whose comments have greatly improved both the focus and quality of this paper. All errors can be blamed on the lack of enforceable commitment mechanisms.
1 Introduction

Many of us would like to exercise, work efficiently and stay away from our bad habits yet we often find ourselves skipping a daily run, looking at funny cat videos on the internet, smoking another cigarette or eating another cupcake. Such mismatches between what we would like to do tomorrow and what we actually end up doing create a demand for technologies to help individuals implement their normative goals. Existing literature\(^1\) shows that this demand for commitment does exist but there is little theoretical work on what forms optimal commitment technologies would take. This paper begins to bridge this gap.

The first step to figuring out optimal commitment technologies is learning what mechanism generates the problem in the first place. Work in psychology and neuroscience tends to focus on decisions as an interplay between an automatic and a controlled process (Kahneman (2003)). Recent work in social psychology makes this model more sharp by positing that the controlled cognitive process uses a limited, costly resource to operate. A standard template in such experiments involves subjects performing a ‘resource depleting task’ (controlling attention, suppressing emotion, solving math problems) or a control task followed by a second resource depleting task. Subjects who are had to perform the depleting task do worse on the second task than controls (see Muraven and Baumeister (2000) for a survey). Another set of experiments involve individuals making choices under cognitive load (for example, subjects are asked to remember a 7 digit number), these individuals then act more impulsively than controls (eg. by choosing more unhealthy foods to eat as in Shiv and Fedorikhin (1999)). In such a model our difficulties at the desert tray come from an interplay of automatic impulses compelling us to eat, and a conscious use of mental resources not to give in.\(^2\)

This paper uses a particular economic model of this process: the dual-self model of Fudenberg and Levine (2006) (henceforth FL). This model is a specific case of a larger set of costly self-control models.\(^3\) In such models


\(^2\)Work such as Hare, Camerer, and Rangel (2009) in neuroeconomics points to a possible neural algorithm for this model in which control networks in the dorsolateral prefrontal cortex modulate ‘overreactions’ by the brain’s reward system.

\(^3\)Gul and Pesendorfer (2001), Dekel, Lipman, and Rustichini (2009), Gul and Pesendorfer (2004), Noor and Takeoka (2010) are highly visible examples in which self-control costs are variable and depend on the amount of adjustment the control process needs to do. Benhabib and Bisin (2005) consider the case where the control process is treated as having a
decisions are a compromise between a ‘temptation’ ranking and a ‘normative’ ranking, with the DM balancing a desire to choose according to his normative preference with a self-control cost of deviating from the temptation ranking.

The FL model imposes specific restrictions on where the disagreement between the temptation and normative preferences comes from: the DM uses a standard time-separable utility function to evaluate consumption streams (or dynamic plans) but the temptation or automatic process discounts the future at a much sharper rate than the DM would like. Thus the DM is tempted to behave impulsively and must use self-control to choose long-run rewards. Because of this structure FL refer to the automatic impulse as the short-run (SR) self and the cognitive control process as the long-run (LR) self. Note that this is a different type of model than those studied in the literature on hyperbolic/quasi-hyperbolic discounting (eg. Laibson (1997), Ainslie and Haslam (1992)). In those models the DM’s problems come from the fact that rankings of alternatives change in different periods. In the FL model both the SR and LR self are perfectly time consistent so the tension comes from multiple preferences within a period rather than multiple preferences between periods. The purpose of this paper is not to provide a clean test to differentiate these models, rather it is to look at commitment behavior with the FL model. However, all results that could never hold under a time-inconsistent framework are flagged as such.

The FL model generates a huge demand for commitment and this paper considers two types of technologies that could be available to a DM facing temptation. The first are stick contracts that levy a fine on the DM when he gives in to temptation. The second are carrots that reward a DM who takes normatively good actions. In this paper, carrots are financed by the intertemporal substitution of future consumption to the present conditional on the DM resisting a temptation. The main results show that both of these types of contracts can only be welfare improving for a DM if they change the nature of the temptation he faces - i.e. the SR self’s optimal action. The logic is one of revealed preference combined with the economic idea that self-control is treated as a cost. If a DM gives in to a temptation, this is because

\footnote{The analysis of Fudenberg and Levine (2006) considers the case where the SR self is perfectly myopic, Fudenberg and Levine (2012) extends to the case where the SR self can have some degree of patience.}

\footnote{A market version of such contracts is provided by the website StickK.com that allows individuals to set measurable goals, punishments and a referee. If an individual does not accomplish his goal, as reported by the referee, the website will automatically charge the individual's credit card a donation to an ‘anti-charity’ of his choice (for example, a life-long Democrat may choose to donate to finance the George W. Bush Presidential Library.)}
the self-control required to resist it was too expensive. If a commitment technology does not physically remove the tempting option, then it must remove the temptation associated with that option because if it does not, its ultimate effect is only to make the DM exert the self-control that he didn’t find optimal to exert in the first place. This means that if the source of the temptation is sharp discounting, both types of contracts must have one of two features: either their effects must be close in time to the choice or they must be particularly large. In fact, as the paper shows later, their size increases exponentially with the delay between action and punishment.

The natural question to ask given these results is whether carrots, sticks or binding commitments are favored by the dual-self DMs. The final set of results shows that carrots are preferred for several reasons. First, sticks will implement a punishment if the DM trembles and executes unintended actions with small probability whereas a tremble in the presence of a carrot makes the DM no worse off than before the contract. Second, sticks and binding commitments require self-control to implement in the first place and if the DM receives opportunities to cancel the contract, the cancellation acts as an additional temptation and source of self-control problems in the case of sticks and binding commitments but not carrots. Finally, the DM may have a desire for flexibility. If the temptation’s size stochastic optimal carrots allow the DM to retain the flexibility choose the temptation when it is LR optimal while sticks and binding commitments do not.

The results in this paper are related to those of Ali (2011) who shows that carrots are advantageous for DMs seeking flexibility in a planner-doer model of self-control in which the planner attempts to learn about the doer’s preferences. From a theoretical perspective, this paper together with the results of Ali (2011) indicate that there is much to be learned by adding dynamic behavior into models of self-control. More practically they also indicate that devices beyond binding commitments and self-punishing technologies may be useful ways of dealing with self-control problems.

2 The Basic Model

We now introduce the basic dual-self model used in Fudenberg and Levine (2012) in continuous time. All formal proofs of results are relegated to the appendix.

The DM begins at time $t = 1$ and faces a ‘simple temptation.’ He chooses an action from the set $\{T(ake), R(exist)\}$. If he chooses $T$ he gains a benefit $b$ at that moment, however, at $t = 2$ he takes a loss of 1. If he chooses $R$, he gains no payoffs but suffers no loses later. Taking a temptation means that
the DM takes a payoff now for a loss later while $R$ keeps payoffs constant between periods.

The DM is split into two ‘selves’ that interact to make decisions. The long-run (LR) self discounts the future with an instantaneous discount rate of $\rho$ (which we take to be 0 for simplicity) and the short-run (SR) self discounts at a faster rate $\lambda$. The intuition behind how the selves interact is as follows: when the DM faces a choice, the SR self ‘suggests’ to take the action that maximizes SR utility, the LR self can then choose to either go with this suggestion or to change to a different action. However, if the LR self wishes to change course he must pay a self-control cost to do so. This cost is proportional to the amount of utility the SR self must give up. Thus, this model represents in a simple, stylized way an interaction between ‘automatic’ or ‘affective’ processes that drive individuals toward immediate rewards (the SR self) and cognitive processes (the LR self) that can be invoked to control them and that become more costly as temptations become larger. For the rest of this exposition, as has been common in the literature, the LR self’s preferences will be used whenever welfare metrics for the DM are discussed.

We now formalize the discussion above: suppose that the DM faces a set of consumption streams $A$, the LR self’s utility from an action $x$ is given by

$$u_{LR}(x) - SC(x)$$

where $u_{LR}(x)$ is the discounted value of $x$ using the discount rate $\rho$ and $SC(x)$ is the self-control cost the LR self must pay to implement action $x$. The self-control cost of taking the SR self’s preferred action $x^*_{SR}$ is 0 and the cost of self control from choosing a different action is given by

$$SC(x) = \psi(u_{SR}(x^*_{SR}) - u_{SR}(x))$$

where $u_{SR}(x)$ is the discounted value of $x$ using the SR discount rate $\lambda$.

Thus, the LR self will use self control whenever the benefit from choosing $x$ instead of $x^*_{SR}$ exceeds the self-control cost required to do so. In the analysis that follows, $\psi$ will be assumed to be a linear function as in Fudenberg and Levine (2006) but the main results in this paper are robust to using a different functional form.

Note that the LR self is perfectly time consistent but does display demand for binding commitment as the mere existence of tempting options creates a potentially costly conflict. Note as well that unlike in models of time-inconsistency, the DM may want to remove options from future choice sets even when he knows for sure that he will not take them.

Applying this model to the simple temptation problem, this interaction is played out in the following way: if $R$ is chosen, the SR self gains a utility
of \( u_{SR}(R) = 0 \) while if \( T \) is chosen the SR self gains utility

\[
u_{SR}(T) = b - e^{-\lambda}.
\]

Suppose that \( b - e^{-\lambda} > 0 \) so the SR self prefers to take the temptation. This means the LR self gets utility 0 from \( R \) and gets utility from choosing \( T \) given by

\[
b - 1 - SC(T).
\]

To motivate the problem, we take \( b < 1 \) and \( -\psi(b - e^{-\lambda}) < b - 1 \). Without any intervention we have that the SR self prefers to take, the LR self prefers to resist but will not do so given the size of self-control costs.

### 2.1 Sticks

We now consider how the DM can implement the long-run optimal action in the absence of perfectly binding commitment devices. First we consider how a DM is able to use contracts that assign a penalty of size \( k \) to a choice of \( T \) (sticks). This penalty is delivered at time \( t' \in [1, 2] \). Intuitively, one can think of this as how long it takes for the contract enforcement mechanism to observe the agent’s action and implement the punishment. Several of the results consider what happens when \( t' > 1 \) but setting \( t' = 1 \) does not change other results. Figure 1 shows the full timeline.

We can now turn to existing sticks for intuition: alcoholics sometimes take the drug Antabuse. This drug stops the body from correctly metabolizing alcohol - the result makes it so that taking even a small drink of alcohol results almost immediately into an experience similar to a severe hangover. Control-It! is a foul tasting (but completely safe) liquid that individuals who are attempting to quit biting their nails apply to them. Individuals trying to lose weight take the drug Xenecal (available over-the-counter as Alli in the United States). This drug acts in a similar manner to Antabuse by making the eating of fats a highly unpleasant experience. In each of these examples punishments come almost immediately after the bad behavior in question and so \( t' \) is very close to 1. However, if we consider StickK.com, punishments are only realized after the DM’s referee reports that he has chosen \( T \). In this case, it seems reasonable to think that \( t' \) will be bounded away from \( t = 1 \) by a non-trivial amount.

In this setup LR utility at the time of choice of choosing to submit to temptation given a stick of size \( k \) is given by

\[
u_{LR}(T, k) = b - k - 1
\]
and the SR utility of the same is given by

$$u_{SR}(T, k) = b - e^{-\lambda' k} - e^{-\lambda}.$$  

To make notation simpler, for an action $\sigma \in \{T, R\}$ and contract of size $k$ we define $\eta(\sigma, k)$ to be the self-control cost associated with taking that action as introduced above. Let $\sigma^*(k)$ be the action that maximizes the LR self’s utility net of self-control costs (and thus the DM’s welfare). We denote the welfare by

$$W(\sigma, k) = u_{LR}(\sigma, k) - \eta(\sigma, k).$$

We can now discuss the effects of the contracts on the DM’s behavior and welfare. First, define:

**Definition 1.** Say that a stick implements resisting if $\sigma^*(k) = R$. Let $K^R$ be the set of sticks that implement resisting.

We now turn to defining optimal sticks:

**Definition 2.** The set of optimal sticks is defined by

$$K^* := \{k \in \mathbb{R}_+ \mid W(\sigma^*(k), k) \geq W(\sigma^*(k'), k') \}$$

for any $k' \in \mathbb{R}_+$.

The next proposition concerns the perfectly myopic SR selves studied in Fudenberg and Levine (2006).

**Proposition 1.** Suppose the SR self completely discounts all future payoffs and $t' > 1$, then $K^R \neq \emptyset$ but $K^* = \{0\}$.

This result states that when the SR self is perfectly myopic, the DM is always made worse off with a stick that does not punish him immediately when he succumbs to temptation. This means that under such circumstances, the DM will never take a stick when offered. The intuition for the proposition comes from the rational way in which self-control is treated in the FL model.
We know that under the condition \( k = 0 \) the cost of self-control to implement resisting was larger than the foregone gains to the LR self. Since at time \( t = 0 \) the whole of the contract is in the future a perfectly myopic SR self is completely unaffected by the imposition of the contract; the only channel that remains for the contract to operate through is making the future consequences of the temptation so unattractive that the LR self will choose to exercise self-control and avoid the extra punishment. Thus the existence of the contract represents a welfare decrease for the LR self. Allowing for SR discount rates below 100% recovers the existence of optimal sticks:

**Proposition 2.** Suppose that the SR self discounts at a positive but not infinite rate, then \( K^* \subsetneq K^R \). Moreover

\[ K^* = \{ k \geq k^* = \frac{b - e^{-\lambda}}{e^{-\lambda} - \psi} \} \]

and so is independent of \( \psi \) and LR discounting.

This proof of the proposition shows that for an FL DM works in a very particular way. Simply implementing resisting is not enough to make an optimal contract, rather the possible loss from the contract must be large enough to change the optimal course of action for the SR self. Note that it is possible to refine this set further using the intuition that there is no incentive for the DM to take a contract beyond the size of \( k^* \) as a larger stick does not change any behavior on the equilibrium path but increases possible losses to the DM in the case of a tremble. This intuition is developed in the Appendix.

There are two other points to be taken from this analysis. First:

**Corollary 1.** For any value of \( \lambda \) self-control is never exercised with any optimal stick.

This implies that optimal sticks can simulate perfectly binding commitments and thus are a useful device for dual-self DMs. Additionally, in the dual-self model, sticks have one additional property:

**Corollary 2.** For any value of \( \lambda \) there exists an open set \( (k, \bar{k}) \) such that \( k \in (k, \bar{k}) \) implements resisting but does not improve welfare over \( k = 0 \).

Note that this is would be impossible in a model of time-inconsistency (assuming that we used the common welfare criterion of the \( t = 0 \) self) as in such a model any commitment device that implemented \( R \) would necessarily be welfare increasing. In addition, this interval can, in general, be quite large as \( k \) is a constant and \( \bar{k} \) strictly increases in \( \lambda \).

The next proposition characterizes a property that all optimal sticks must display:
Corollary 3. The lower bound for optimal sticks $k^*$ grows exponentially as the enforcement time, $t'$ moves away from 1. The speed of this growth is proportional to the SR discount rate $\lambda$.

While in a perfect world this proposition poses no problems, in a world where DMs can tremble to Take with a small probability there is a completely different story. When contracts can be taken that employ punishment almost immediately, their effects do not have to be extraordinarily large but as we push the time of resolution backward and the contract grows exponentially we run into problems.

To appreciate the impact of this growth, consider a case where $b = .5$ and that the SR self discounts at a rate of 50% per day with $t = 2$ approximately 1 month later. If $t'$ is three days later the optimal stick size is approximately $k = 4 –$ this is 8 times the gain from the contract. If $t'$ is a week later then the optimal contract has a punishment size of 64 times the utility gain from implementing the normatively superior action. In this case, even a 2% chance of the DM trembling to adhere to the long-run optimal action makes the contract not worth it. A similar result can be recovered if we require simply that the stick is welfare improving and not necessarily optimal.

2.2 Carrots

We now consider a different type of commitment contract, a carrot. In this type of contract the DM receives a reward of utility size $r$ if he takes the long-run optimal action. For simplicity, we assume that this reward is financed by borrowing from future consumption. For example, a DM may commit to ‘buying themselves something nice’ if they manage to finish a particular project on time. Additionally, to make the comparison between the costless sticks and carrots possible we assume that the LR preference is indifferent between consuming $r$ today and the discounted future value of $r$.

The timeline with carrots is identical to that of the one with sticks: the carrot is administered at time $t' \in [1, 2]$ if the long-run optimal action $R$ is

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6Pinning down SR discount rates is an important but difficult empirical challenge. For example, McClure, Ericson, Laibson, Loewenstein, and Cohen (2007) consider the case of primary rewards (juice for thirsty individuals) and show that the discount horizon for the ‘short-run’ is about $\sim 60\%$ per 25 minutes in their experiment. However in many other experiments (including any ones that involve monetary payouts) individuals appear to react impulsively to short run rewards that they will not receive until at least the end of the experiment. Clearly, SR discounting must display some sort of context dependence.

7If this neutrality isn’t satisfied then the intertemporal loss from changing consumption can be viewed as a shadow cost of using the carrot contract even in the absence of other costs. Thus, the DM will only take a carrot if the intertemporal loss is less than $1 – b$. The appendix discusses in more detail the magnitude of these shadow costs.
taken. If the DM chooses $T$ then the carrot is not activated and the future consumption stream of the DM is untouched. We maintain identical notation from the last section and define the set of resistance implementing carrots $\mathcal{R}^R$ and the set of optimal carrots $\mathcal{R}^*$ analogously.

In the case of perfectly myopic SR selves we get an analogue to proposition 1 for carrots:

**Proposition 3.** Suppose the SR self completely discounts all future payoffs and $t' > 1$, then $\mathcal{R}^R = \emptyset$ and $0 \in \mathcal{R}^*$

Thus carrots are never useful when the SR self perfectly discounts the future. However, when the SR self has non-negligible valuation of future consumption carrots become useful:

**Proposition 4.** Suppose that the SR self discounts at a positive but not infinite rate, then $\mathcal{R}^* \subsetneq \mathcal{R}^R$. Moreover

$$\mathcal{R}^* = \{r \geq r^* = \frac{b - e^{-\lambda}}{e^{-\lambda t'}}\}$$

and so is independent of $\psi$ and LR discounting.

Finally, just like sticks, carrots are able to perfectly simulate binding commitment:

**Corollary 4.** For any value of $\lambda$ self-control is never exercised with any optimal carrot.

And finally just like sticks:

**Corollary 5.** The lower bound for optimal carrots $r^*$ grows exponentially as the enforcement time, $t'$ moves away from 1. The speed of this growth is proportional to the SR discount rate $\lambda$.

Thus, sticks and carrots are useful replacements for DMs seeking binding, but unavailable, commitment devices.

### 3 Comparison

We now turn to comparing the welfare effects of carrots to sticks. Notice that if we assume that the DM trembles to the unintended action with probability $\epsilon$, carrots mechanically gain an advantage over sticks because they provide no downside risk. We now consider two more factors that could be at play in real commitment decisions. First, we add the presence of stochastic opportunities
to ‘call off’ a commitment. Second, we consider the role of flexibility in commitment. We show that each of these factors weigh the welfare scale towards carrots and away from both sticks and binding commitments.

We consider a DM who faces the simple temptation above at time \( t = 1 \) but, at \( t = 0 \), has a chance to choose a commitment device from the menu \( \{ r^*, k^*, s, N \} \) where \( N \) is no commitment, \( k^* \) is the optimal stick, \( r^* \) is the optimal carrot and \( s \) is the binding contract that removes \( T \) from the DM’s menu. We assume that the SR self is not perfectly myopic (thus \( k^* \) and \( r^* \) can simulate binding commitment even if \( t' > 0 \)).\(^8\) This is the key assumption that drives the revision result but not the flexibility result.

Furthermore we assume between \( t = 0 \) and \( t = 1 \) the DM lives in continuous time and receives opportunities to revise his choice and costlessly replace any current commitment with \( N \). These opportunities arrive as a Poisson process with arrival rate \( \mu \in [0, \infty) \). Figure 2 shows the structure of the problem.

![Timeline of the expanded problem](image)

**Figure 2:** Timeline of the expanded problem.

To solve the problem the DM faces, we must specify how the SR self evaluates future actions at \( t = 1 \) from a \( t = 0 \) perspective. Here we follow the assumption in Fudenberg and Levine (2012) that the SR self is strategically naive and assumes that all future decisions will include no exercise of self-control by the LR self.\(^9\) We get the following characterization of the DM’s behavior:

**Proposition 5.** For any value of \( \mu \), the DM strictly prefers choosing the optimal carrot at time 0 to any other option. Furthermore, there exists \( \overline{\mu} \) such that if \( \mu > \overline{\mu} \) the DM strictly prefers no commitment to taking the optimal stick or the binding commitment device.

\(^8\)Fudenberg and Levine (2012) give many reasons for assuming that the SR preferences display some degree of patience. Rather than recapitulate their arguments, we point the interested reader to that paper.

\(^9\)Note that we have already shown that no self-control is ever exercised at optimal carrots/sticks so the main results in this section could still be derived under the assumption that SR self is strategically sophisticated.
The intuition for the proposition comes from how sticks and carrots simulate binding commitment in different ways. Optimal sticks and carrots both make the SR self indifferent between $T$ and $R$, however they do so in different ways. The stick lowers the utility of the temptation for the SR self while the carrot raises the utility of resisting. This means that the SR self is made worse off in the future by either binding commitment or the use of a stick but is made weakly better off by the use of a carrot. The SR self isn’t perfectly myopic and so now prefers carrots and no commitment to sticks and binding commitments. Because of these SR preferences taking a stick or binding commitment requires self control by the LR self when $N$ is in the choice set, but taking a carrot does not.

The same logic holds during revision opportunities: canceling a stick or binding commitment is itself a temptation that the DM must use self-control to resist. Thus, if the DM expects many revision opportunities before $t = 1$ he may find that the expected self-control of taking a stick or binding commitment cost exceeds the benefit. Contrarily, canceling a carrot is not tempting and so the presence of revision opportunities does not affect the DM’s valuation.

3.1 Flexibility

Another reason for the preference towards carrots over sticks comes from a desire for flexibility. We can expand the model above to one where $b$ (the temptation) payoff is not a single number but is in fact stochastic and ex-ante unknown. Suppose that $b$ is distributed on $[0, b_{\text{max}}]$. We suppose this distribution is well behaved and we call the PDF $f(\cdot)$ and the CDF $F(\cdot)$.\textsuperscript{10} We now ask how demand for sticks, carrots and binding commitments changes in this new situation. As above, suppose that sticks/carrots/binding commitments are taken at time 0 and the DM is aware of the distribution of $b$ at time 1. We set the revision parameter to 0 to isolate flexibility motives.

We now compute the DM’s expected welfare from being uncommitted, then compare that to the DM’s welfare from having a binding commitment, a stick contract or a carrot contract. Note that when the DM is uncommitted due to continuity of the LR welfare in $b$ we can break up the interval $[0, b_{\text{max}}]$ into several components. We know there exists a subinterval $[0, \tilde{b}]$ in which the DM resists the temptation and exercises no self-control (as the SR self does not prefer $T$ to $R$). We also know that there exists another subinterval $[\tilde{b}, \bar{b}]$ such that for any value of $b$ in this interval, the DM resists the temptation (but pays a self-control cost to do so). In the interval $[\bar{b}, 1]$ the DM does not

\textsuperscript{10}We do not require full support assumptions.
resist the temptation (but is made worse off), and finally, in the interval 
$[1, b_{\text{max}}]$ the DM chooses $T$ and because it is LR superior to $R$.

We can compute the borders for these intervals simply: the first bound is the SR self’s indifference point $b = e^{-\lambda}$ and the second is the level of $b$ at which the DM is indifferent between taking and resisting

$$b^* = \frac{1 + \psi e^{-\lambda}}{1 + \psi}.$$

We can compute that the DM is made better off under a binding commitment that under no commitment if and only if the following condition holds:

$$\int_{b^*}^b \psi(b - e^{-\lambda})dF + \int_{1}^{b^*} (b - 1)dF \geq \int_{b_{\text{max}}}^{b^*} (b - 1)dF.$$

The intuition behind this condition is the following: the left hand side of the inequality represents the gains from commitment. If $b$ falls in the interval $b$ to $\tilde{b}$ and he were uncommitted, the DM would have to exercise self-control. If $b$ falls in the interval $\tilde{b}$ to 1 the DM would give in to the temptation. Having a binding commitment removes these two problems. However, it comes at a cost. If $b$ falls in the interval 1 to $b_{\text{max}}$ the DM would like, from an LR perspective, to chose $T$. Thus, in this case a binding commitment causes the DM to lose potential payoff.

Now, suppose we also allow the DM to choose from a full menu of sticks and carrots (that is, any values of $r$ or $k$). We get the following proposition:

**Proposition 6.** For any distribution of temptations $F(\cdot)$ the DM always weakly prefers sticks to binding commitments. Moreover, there exist distributions such that this preference is strict. Additionally, for $F(\cdot)$ that puts positive weight on any $b > 1$ there exists a carrot such that DM is always strictly better off under that carrot than under any stick or binding commitment.

This proposition comes from the intuitions developed in the last section. First, sticks are at least as valuable as binding commitments because a very large $k$ can always exactly simulate the binding commitment. This can also hold strictly: the proof shows an example in which sticks yield a strictly higher welfare than binding commitments.

The final part of the proposition comes from the fact carrots have no downside: if the DM chooses $T$ under a carrot contract he simply foregoes his separate reward for $R$, however once a DM has chosen a stick he must lose payoff whenever he chooses $T$. Thus, carrots allow for maximum flexibility and are strictly preferred by the DM whenever flexibility is possible.
4 Conclusion

Models of self-control problems all predict a large demand for commitment in our daily lives and indeed these technologies are all around us. Economists have studied binding financial commitment devices such as Christmas Clubs (Loewenstein and Thaler (1989)) or the use of illiquid assets (Laibson (1997)). In the digital world, the Apple Mac App store contains apps such as Concentrate or Self-Control that allow the user to turn off access to certain websites (e.g. Facebook) during work hours. However, though these devices exist, the predicted demand of commitment seems to far outstrip the much more limited supply.

Existing literature gives one reason for this dearth: a lack of sophistication (DellaVigna and Malmendier (2006)) on the part of DMs. On the other hand, others have argued that combined with learning, an initial lack of sophistication can only lead to overcommitment in the long run (Ali (2011)). The results in this paper show (as others, e.g. Bryan, Karlan, and Nelson (2010), have informally stated) that there may be many other good reasons for individuals not to want binding commitments or sticks even when they know that they will be faced with temptations. In addition, at least anecdotally, it seems that many individuals do use a form of carrots to motivate themselves to fight temptations such as procrastination (for example, agreeing to go out to a nice meal with friends conditional on finishing a work project) so actually these types of commitments may be quite prevalent and understudied.

Many interesting issues remain in the study of self-control – especially in the domain of dealing with self-control problems. This paper is meant to be a first step in applying the existing economic theories to both the understanding how individuals deal with self-control problems and the task of designing useful mechanisms that individuals could use to stick to their long-run optimal plans.
References


5 Appendix 1: Strictly Optimal

Suppose that we consider a world where the DM is restricted to trembling to the action $T$ with probability $\epsilon$. Assume that this tremble incurs no self-control cost (the next section of the appendix discusses in more technical detail why we use this formulation instead of a formulation where the control cost is allowed to depend on the tremble probability) so that a DM who trembles to $T$ receives final LR utility $b - k - 1$. Let

$$ W_\epsilon(\sigma^*(k), k) = (1 - \epsilon)W(\sigma^*(k), k) + \epsilon(b - k - 1). $$

This lets us define a stronger notion of optimality for stick contracts.

**Definition 3.** Say that $k$ is a **strongly optimal KM contract** if for any $k' \in \mathbb{R}_+ \setminus k$ there exists a sequence $\epsilon_n \to 0$ such that

$$ W_{\epsilon_n}(\sigma^*(k), k) > W_{\epsilon_n}(\sigma^*(k'), k) \text{ for each } n \in \mathbb{N}. $$

Let $K^{**}$ be the set of strongly optimal KM contracts.

The logic behind this definition is a sort of purification argument similar to that employed in many game-theoretic refinements. The next proposition shows that the definition has bite as a refinement of the optimal stick set:

**Proposition 7.** For an FL DM with $\lambda < \infty$ we have that $K^{**}$ is a single point given by

$$ k^* = \frac{b - \gamma}{e^{-\lambda t'}}. $$
6 Appendix 2: Mixed Strategies

We may want to model not exogenously specified trembles but consider DMs who make choices that are not simply single elements of some choice set but probability distributions over the choice set. To analyze such choices, we need to make assumptions on how the choice of probabilistic actions influences the self-control costs of the LR self. In this section we will relax the linearity assumption made in the body of the paper and let $\psi$ be a general weakly convex, smooth function with $\psi(0) = 0$.

For notation, let $M(A)$ be the set of probability distributions over a finite set of alternatives $A \subset \mathcal{X}$ with generic element $\sigma$. Let $\sigma(y)$ be the probability that $\sigma$ assigns to $y \in A$. There are two main ways to consider:

**Definition 4.** Fix $\sigma \in M(A)$, the *ex-ante expected self-control cost* denoted $\zeta(\sigma, A)$ of $\sigma$ is given by

$$\zeta(\sigma, A) = \psi(\bar{u}_{SR}(A) - \sum_{y \in A} \sigma(y)U_{SR}(y))$$

The intuition behind this formulation is that the LR self actually chooses a probability distribution over elements of $A$, pays the self control cost for the distribution and then realizes some draw from the distribution. The other formulation is as follows:

**Definition 5.** Fix $\sigma \in M(A)$, the *ex-post expected self-control cost* denoted $\eta(\sigma, A)$ of $\sigma$ is given by

$$\eta(\sigma, A) = \sum_{y \in A} \sigma(y)\psi(\bar{u}_{SR}(A) - U_{SR}(y))$$

In this formulation the randomization is viewed as one over the LR self’s control actions. Here the LR self uses the randomizing device to select an action but pays self-control costs to execute it as in the standard definition.

For simplicity we now assume that we have for all $A \subset \mathcal{X}$ and $y \neq y' \in A$

$$U_{LR}(y) - \psi(\bar{u}_{SR}(A) - U_{SR}(y)) \neq U_{LR}(y') - \psi(\bar{u}_{SR}(A) - U_{SR}(y'))$$

so the LR self is never indifferent between any two options when self-control costs are taken into account. Then fix $A \subset \mathcal{X}$ and let $\sigma^*_{\eta}$ be the solution to

$$\max_{\sigma \in M(A)} \sum_{x \in A} \sigma(x)U_{LR}(x) - \eta(\sigma, A)$$

and $\sigma^*_{\zeta}$ be the solution to the same problem using the ex-ante self-control formulation.

The choice of formulation is not without consequences:
Proposition 8. For any \( A \subset \mathcal{X} \) we have that \( \sigma^*_n(x) \in \{0, 1\} \) for any \( x \in A \).

Proof. Recall that the LR utility of choosing mixed strategy \( \sigma \) is given by

\[
\sum_{y \in A} \sigma(y)U_{LR} - \sum_{y \in A} \sigma(y)\psi(U_{SR}(x^*(A)) - U_{SR}(y)).
\]

But this is just

\[
\sum_{y \in A} (\sigma(y)U_{LR} - \sigma(y)\psi(U_{SR}(x^*(A)) - U_{SR}(y))).
\]

By assumption of no indifference there exists a unique \( y \in A \) to maximize this.

Thus under the ex-post formulation a DM who is not constrained to choose a distribution with full support always makes a deterministic choice. This is not the case for the ex-ante formulation. Consider an example where utility is linear and the choice set \( A \) is given by the consumption streams \( x = (1, 0) \) and \( y = (0, 2) \). Suppose further that \( \gamma = 0, \delta = 1 \) and \( \psi(z) = az^2 \).

The LR utility of choosing a distribution \( \sigma \in \mathcal{M}(A) \) is given by

\[
\sum_{x \in A} \sigma(x)U_{LR}(x) - \zeta(\sigma, A)
\]

that in this case is

\[
2(\sigma(y)) + (1 - \sigma(y)) - a(1 - (1 - \sigma(y)))^2.
\]

It can be readily checked that the optimal solution sets

\[
\sigma(y) = \frac{3}{2a}
\]

for values of \( a > \frac{3}{2} \) and thus is interior. However, this effect is a result of the choice of self-control cost function:

Proposition 9. If \( \psi \) is a linear function then

\[
\eta(x, A) = \zeta(x, A)
\]

for any choice of \( A \subset \mathcal{X} \) and \( x \in A \).

If we are not restricting \( \psi \) to be linear and since the choice of non-degenerate randomizations in basic maximization problems seems to be intuitively unappealing so the ex-post self-control formulation seems more reasonable.
Appendix 3: Proofs of Propositions

We first prove an auxiliary lemma that makes the following results much simpler.

**Lemma 1.** The function $W(\sigma^*(k), k)$ is continuous in $k$.

*Proof of Lemma 1.* Fix $k$. If there exists a neighborhood $N$ of $k$ such that $\sigma^*(k) = \sigma^*(k')$ for all $k' \in N$ then clearly $W(\sigma^*(k), k)$ is continuous at $k$.

To look at the kink points of $W$ first notice the following basic property: $W(T, k)$ is decreasing in $k$ and $W(R, k)$ is weakly increasing and both are continuous. Therefore, there exists $k^*$ such that $\sigma^*(k) = T$ for all $k < k^*$ and $\sigma^*(k) = R$ for all $k \geq k^*$. But at $k^*$ we have that $W(T, k^*) = W(R, k^*)$ so $W(\sigma^*(k), k)$ is continuous at $k^*$ also.

*Proof of Proposition 1.* The fact that $\mathcal{K}^R$ is non-empty is obvious (just consider arbitrarily large $k$).

For the second part of the proposition, suppose the DM has $\gamma = 0$ then $SC(R, k) = \psi(b) = SC(R, k')$ for all $k, k' \in \mathbb{R}_+$. This means that $W(R, k) = W(R, k') < W(T, 0)$ for all $k, k' \in \mathbb{R}_+$. But since $W(T, k)$ is decreasing in $k$ it is the case that for any $k > 0$ we have that

$$\max\{W(T, k), W(R, k)\} < W(T, 0).$$

Thus $\mathcal{K}^* = \{0\}$. 

*Proof of Proposition 2.* To show this look at the utility of choosing $R$ with stick contract of size $k$, this is simply

$$-\psi(b - e^{-\lambda} - e^{-\lambda'} k)$$

if $T$ is preferred to $R$ by the SR self and 0 otherwise. $k^*$ thus sets $SC(R, k^*) = 0$ which is exactly when

$$e^{-\lambda'} k = b - e^{-\lambda}.$$

Algebra gives:

$$k^* = \frac{(b - e^{-\lambda})}{e^{-\lambda'}}.$$

Corollary 1 follows from this argument as well. Note that the fraction has a denominator less than 1 which shrinks exponentially in $t'$, thus $k^*$ grows exponentially in $t'$ and this shows Corollary 3.

20
Proof of Corollary 2. Fix a DM by the argument in the proof of Proposition 1 there exists a $k$ such that $k \geq k$ implements resisting. By the indifference argument from the proof of Proposition 1 it must be that

$$W(0, P) > W(k, I) = W(k, P)$$

however by continuity it must be that for a small neighborhood $(k, \bar{k})$ it is the case that

$$W(0, P) > W(k', P) \forall k' \in (k, \bar{k}).$$

But each $k'$ is also an element of $\mathcal{K}^R$ by construction, thus we have proved the corollary.

Proof of Proposition 3. Note that for any $r$ implemented at $t' > 1$ does not affect the preferences of the perfectly myopic SR self. By the way the carrot is financed, the LR self is also completely indifferent between the case where $r$ is given at $t'$ and lifetime consumption changes to finance the carrot or whether the carrot is not activated. In this case, no carrot can affect decisions.

Proof of Proposition 4. To show this proposition, call the utility of the SR self from taking action $x$ under a carrot of size $r$ to be $u_{SR}(x, r)$. Then

$$u_{SR}(T, r) = b - e^{-\lambda}$$

while

$$u_{SR}(R, r) = e^{-\lambda't'}.$$ 

This means that the self-control cost of $R$ can be written as

$$\psi(b - e^{-\lambda} - e^{-\lambda't'})$$

while $u_{SR}(T, r) > u_{SR}(R, r)$ and 0 afterwards. This cost decreases continuously to 0 in $r$ so that means there must exist an $\tau$ such that for $r > \tau$ the

$$b - 1 > -\psi(b - e^{-\lambda} - e^{-\lambda't'})$$

so the LR self chooses $R$. Thus

$$\mathcal{R}^R = \{r \in R \mid r \geq \tau\}.$$ 

This also means that we can take

$$r^* = \frac{b - e^{-\lambda}}{e^{-\lambda't'}}$$

and we have that $r \geq r^*$ means the SR self weakly prefers $R$ to $T$ and thus that self-control costs are 0 and the DM chooses $R$. From this argument, combined with the one proving Proposition 3 we have that Corollary 4 is obvious. From the expression we can also see that Corollary 5 holds.
Proof of Proposition 5. We can now consider the case of accepting a stick of size $k^*$. The SR self’s utility of choosing $k^*$ at time 0 is given by $u_{SR}(k^*, t = 0) = 0$ because the SR self correctly anticipates that $R$ will be taken at time $t = 1$. Notice that this means that the SR’s utility of choosing $N$ at $t = 0$ is given by
\[ u_{SR}(N, t = 0) = e^{-\lambda}(1 - e^{-\lambda}) \]
which is greater than 0. Thus the self control cost from taking $k^*$ from any menu that includes $N$ is strictly positive. Note that an identical analysis applies to choosing a binding commitment when the option for no commitment is present.

Now consider the case of taking $r^*$ at $t = 0$. Under a carrot of $r^*$ the SR self is exactly indifferent between choosing $R$ and $T$ at $t = 1$. This means that
\[ u_{SR}(c^*, t = 0) = e^{-\lambda}(1 - e^{-\lambda}) = u_{SR}(N, t = 0). \]
Thus the self control cost of implementing $r^*$ at $t = 0$ is zero and so the DM strictly prefers carrots to sticks and binding commitments.

Now, suppose that the agent gets a revision opportunity at some time $t \in (0, 1)$. If faced with a carrot, he has no incentive to revise. If faced with a stick or binding commitment, choosing $N$ now carries an SR utility of
\[ u_{SR}(N, t = t) = e^{-\lambda(1-t)}(1 - e^{-\lambda}) > u_{SR}(N, t = 0) \]
and thus a self control cost of $\psi(u_{SR}(N, t = t))$.

The ex-ante expected utility of taking a binding commitment or $k^*$ at $t = 0$ given a Poisson arrival rate $\mu$ is continuously decreasing in $\mu$ and bounded above by $-(1 + \mu)\psi(u_{SR}(N, t = 0))$ which decreases without bound. Thus there exists an $\overline{\mu}$ such that for $\mu > \overline{\mu}$ the DM prefers, at $t = 0$ to have no commitment than either $k^*$ or a binding commitment. Note that because there is no temptation to revise a carrot, the expected welfare of a DM with a carrot contract is constant in $\mu$. \qed

Proof of Proposition 6. Suppose that we set $k > b_{\text{max}} e^{-\lambda \mu}$ then for any realization of $b$ the SR self prefers $R$ to $T$ so a stick can simulate binding commitment (thus the DM is at least indifferent to binding commitments). Now we show that this preference can be strict. Suppose that $b$ can either be $b < b_L < 1$ with probability $p$ or $b_H > 1$ with probability $1 - p$. Thus the ex-ante expected utility of being uncommitted
\[ p(b_L - 1) + (1 - p)(b_H - 1). \]
Now, suppose we take
\[ k = \frac{b_L - e^{-\lambda}}{e^{-\lambda t}}. \]
This means that at \( b_L \) the SR self is indifferent between \( R \) and \( T \). The expected utility of taking \( k \) is then given by
\[ p(0) + (1 - p)(b_H - k - 1). \]
Note that if \( b_H \) is very high then the DM is better off in this contract then under a binding commitment which delivers an expected utility of 0. Note also that given this problem we can set \( p \) to be close to 1 (in which case the ordering is \( \text{stick} \succ \text{binding} \succ \text{no commitment} \)) or close to 0 in which case the order is \( \text{stick} \succ \text{no commitment} \succ \text{binding} \).

Suppose now that we set \( r = \frac{1 - e^{-\lambda}}{e^{-\lambda t}}. \) This means that for all \( b \leq 1 \) the SR self prefers \( R \) to \( T \). However, for all \( b > 1 \) the SR still prefers \( T \) to \( R \) so now LR and SR preferences are aligned in all cases. This means for any \( b \) the DM exerts no self-control and takes the choice that is LR optimal. Thus, carrots make the DM better off than either sticks or binding commitments. \( \square \)
8 Appendix 4: Costs of Carrot Contracts

In the main text, carrots were funded by moving future consumption from future periods to time $t'$. We made the assumption that carrots were costless to make the comparison to costless sticks and binding commitments easier. We now discuss what the true cost of a carrot contract is: we will do this by bounding a DM’s utility loss from perturbing a consumption stream in a particular way. We then calibrate this loss making an assumption of log utility and show that these intertemporal losses are quite reasonable and so even factoring this cost carrots remain a useful tool for DMs seeking commitment.

We look at the effects of taking a carrot on the welfare of the LR self. To do this, we simply consider a normal DM who lives in discrete time, discounts with rate $\delta$ and has a smooth, concave utility function $u$ and has a flat consumption path $(c, c, c, \ldots)$ until the end of time. Now, suppose that this DM is to move $B$ units of consumption from the future into period $t = 1$. He does this in the following way: from each period $t = 2$ to $2 + N$ he borrowers $\frac{B}{N}$ units of consumption.

We now ask, what is the loss from $t = 2$ on from implementing this consumption path. This is exactly

$$\sum_{t=2}^{t+N+1} \delta^{t-1}(u(c) - u(c - \frac{B}{N})).$$

However, note that for a very patient DM and large $N$ this loss can be well approximated by

$$Bu'(c).$$

Formally, the following is true:

**Proposition 10.** For any $\epsilon > 0$ there exists $\delta < 1$ and $N$ such that

$$| \sum_{t=2}^{t+N+1} \delta^{t-1}(u(c) - u(c - \frac{B}{N})) - Bu'(c) | < \epsilon.$$

**Proof.** To show this note that we can use the fact that marginal utility is decreasing to bound this loss by

$$\sum_{t=2}^{t+N+1} \delta^{t-1} \frac{B}{N}u'(c - \frac{B}{N}).$$

This is because for each unit of consumption he loses, the DM loses at most $u'(c - \frac{B}{N})$ units of utility.
Now we can take $N$ to be very large, and thus this is well approximated by
\[
\sum_{t=2}^{t+N+1} \delta^{t-1} \frac{B}{N} u'(c).
\]
Now we can also take $\delta$ to be very close to 1 in which case this is now well approximated by
\[
\frac{NB}{N} u'(c)
\]
which is exactly the result we want.

Now, $Bu'(c)$ is an upper bound for the loss of a very patient LR self from $t = 2$ and on. There is an extra portion to be considered which is the fact that the DM gets to eat these $B$ units at time $t = 1$. At $t = 1$ his gain is bounded below by $Bu'(c + B)$, again due to concavity. This means the total loss from the $B$ perturbation of the DM’s consumption stream is bounded above by
\[
B(u'(c) - u'(c + B)).
\]

We now consider whether this loss is relatively big or relatively small. To do this, we make a functional form assumption on the DM: we say that he has log utility. We also parametrize the size of $B$ in terms of his daily consumption $c$ so $B = rc$. For log utility, the LR loss in utils of the intertemporal disturbance is given by
\[
rc\left(\frac{1}{c} - \frac{1}{(1+r)c}\right).
\]
The expression above simplifies to
\[
rc\left(\frac{rc}{c^2(1+r)}\right)
\]
which further simplifies to
\[
\frac{r^2}{1+r}.
\]
Now we can consider the relative cost of the intertemporal move $B$. Suppose that daily consumption $c$ is $\$100\text{ per day}$, and that $B$ is one day’s consumption (a very nice dinner compete with apertifs, paired wine and a delicious desert). This means that $r = 1$ and so the loss in utils of the move $B$ is $\frac{1}{2}$. Note that daily utility is $log(100) \approx 4.6$ so the shadow cost (in utils) of a carrot of the same size as a full day’s consumption is approximately 10% of a day’s consumption in this case. The table below shows shadow intertemporal costs in terms of the utility of a day’s consumption for various levels
of $B$ and $c$ under the assumption of log utility. Note again that this is an upper bound on losses which may not always particularly tight because for extremely large values of $B$ relative to $c$ as our approximation of the gain at $t = 1$ will be off by quite a bit. However, for the values of $c$ and $B$ considered in the table the bound is relatively tight.

<table>
<thead>
<tr>
<th></th>
<th>$B = $50</th>
<th>$B = $100</th>
<th>$B = $200</th>
<th>$B = $400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = $50</td>
<td>12%</td>
<td>34%</td>
<td>81%</td>
<td>181%</td>
</tr>
<tr>
<td>$c = $100</td>
<td>3.6%</td>
<td>10%</td>
<td>29%</td>
<td>69%</td>
</tr>
<tr>
<td>$c = $200</td>
<td>.9%</td>
<td>3.14%</td>
<td>9.4%</td>
<td>25%</td>
</tr>
<tr>
<td>$c = $400</td>
<td>.2%</td>
<td>.8%</td>
<td>2.7%</td>
<td>8.3%</td>
</tr>
</tbody>
</table>

**Figure 3:** Upper bound on shadow cost of a carrot of size $B$ by levels of daily consumption as percentage of 1 day’s consumption utility.

Thus, reasonably sized carrots (relative to daily consumption) are not that expensive to implement.