Ensemble Learning for Reectometry
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2010)
we describe a variational Bayesian approach to inferring material
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This technical report provides additional detail.
so of that approach.

First, we detail the prior probability distribution.
for all righting reasons.

Second, we provide an overview of
some of the main models of
regulation.
the bilinear likelihood expression that is based on discretizing the re
ndering equation.

Third and

nally, we provide the update equations for
the iterative algorithm that computes an approximation to the posterior
ob ability distribution on the available coefficients of real-world simulation. The mixing weights, $w_m$, are defined in Sect. 3.1 [3]. All coefficients in any one group (defined in Sect. 3.1 [3]) share the same $N_m$, $n_m$, and $p_{nm}$. The exact forms of the mixture components are as follows:

$$p' = Y_{m=2}N_{M}^{N_{m}}p_{nm}(m); (1)$$

with $N_m$ the number of mixture components for coefficient $m$, and the mixing weights. All coefficients in any one group (defined in Sect. 3.1 [3]) share the same $N_m$, $n_m$, and $p_{nm}$. The exact forms of the mixture components are as follows:

$$p_{nm} = (N(0;\nu); \text{if } m \neq 3 N_{RC_{nm}}(\nu_{nm}; T); \text{if } m = 3; (2)$$

where $\nu_{nm} = \frac{v_{nr}(m)}{g(m)}$ if $r(m) \notin \{2, 3\}$; and $\nu_{nm} = \frac{v_{nr}(m)}{g(m)}$ if $r(m) \in \{4, 5\}$. $N_m$ is $\frac{m}{3}$ if $r(m) \in \{3, 4, 5\}$; and $1$ if $r(m) \in \{1, 2\}$.

$$V_{nm} = \begin{cases} 8 \nu_{nm} g(m) & \text{if } r(m) = 2(2; 3)\nu_{nm}(m); \text{if } r(m) = 2(3; 4; 5)g(m) \end{cases}$$

where $\nu_{nm}$ is $\frac{v_{nr}(m)}{g(m)}$ if $r(m) \notin \{2, 3\}$; and $\frac{v_{nr}(m)}{g(m)}$ if $r(m) \in \{4, 5\}$. $N_m$ is $\frac{m}{3}$ if $r(m) \in \{3, 4, 5\}$; and $1$ if $r(m) \in \{1, 2\}$.
categories to which groups (spatial location and basin type) code belongs.
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Letting the rectification point be $T$, say, the rectified Gaussian distribution has the form:

$$p(x) = \begin{cases} 
\frac{2}{p} \text{erfc}(\frac{(u-T)p}{2\sqrt{2}w}) 
& \text{if } x \geq T \\
0 & \text{otherwise}
\end{cases}$$

Furthermore, if $x \sim \text{NRC}(u;w;T)$, then

$$hxi = p(\xi) \frac{2}{p} \text{erfc}(\frac{(u-T)p}{2\sqrt{2}w})$$

2 Discrete rendering equation

We define the following imaging model (Eq. 7 in [3]):

$$I = Z \int L(\theta) V(r(\theta)) F(\theta) \; p(\theta) \; d\theta$$

With $L = P$ and $F = \delta$ (the delta basis in a discretized local hemisphere (a discretization of in Eq. 5), $R$ is the linear transformation that maps from the wavelet basis in domain of the octahedral map into the delta
basis in (see [4]), and Wis the linear transformation that maps from the NMF basis into the delta basis in \( D \). We also represent the visibility in the delta basis in the discrete hemisphere \( (V^d)^P(\Omega) \), and substitute these into the rendering equation: \( \text{df} \)
which can be written in matrix form as \( Rf = \mathbf{M}f \), where the per-pixel matrices \( M_i \)
are given by \( M_i = \mathbf{W}^T \mathbf{R} \mathbf{C} \mathbf{v} \mathbf{W}_i \mathbf{M}_i \mathbf{k}_i \mathbf{w}_i \); (9)

\[ f^+ = (\mathbf{W}^T \mathbf{R}^{-1} \mathbf{C}) f, \]

where \( \mathbf{W} \) is the Hadamard (or entrywise) product, and \( \mathbf{diag}(\cdot) \) is a square matrix
with its argument along the diagonal. Substituting this expression into Eq. 5, we have

\[ f^+ \quad (8) \]

which is exactly Eq. 8 in [3].

### 3 Update equations for the posterior approximation

As mentioned in Sect. 3.4 of [3], the ensemble of distributions \( q(\cdot) \) that
approximate the posterior distribution of reectance \( f \), lighting \( \ell \), and exposure
and noise parameters \( \sigma \) are of the forms

\[ q(m'; m) \quad \text{with} \quad q(m; m) = N(u_m) \quad \text{if} \quad m \neq 3 \quad N_{R,C,V,w}(u; w) \]

\[ q(2) = N(u_2; a_2, b_2) \quad \text{if} \quad m = 3 \quad N_{R,C,V,w}(u; w) \]

\[ q(\cdot) = \prod_{i=1}^{N} q(\cdot) \quad \text{with} \quad q_k \quad N(u_k, m_k; \sigma) \quad \text{otherwise}, \]

\[ m^m = N(u_m) \quad \text{if} \quad m \neq 3 \quad N_{R,C,V,w}(u; w) \]

\[ = \sum_{i=1}^{N} \mathbf{W}_i \mathbf{M}_i \mathbf{k}_i \mathbf{w}_i \]

\[ u_k \quad w_k \]

\[ N(\cdot)^{-1} = \prod_{i=1}^{N} q(\cdot) \quad \text{with} \quad q_k \quad N(u_k, m_k; \sigma) \quad \text{otherwise}, \]

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\[ N(\cdot)^{-1} = \prod_{i=1}^{N} q(\cdot) \quad \text{with} \quad q_k \quad N(u_k, m_k; \sigma) \quad \text{otherwise}, \]

\[ = \sum_{i=1}^{N} \mathbf{W}_i \mathbf{M}_i \mathbf{k}_i \mathbf{w}_i \]
\[
\frac{T}{q(2)} \frac{T}{q(2)} M f \quad ; \quad (15)
\]

\[
2 = 2 \quad q(2)
\]

\[
u = 2 q(2) \quad 2 \prod_{i=1}^{N+i} X_{2i}^{q(2)}
\]

\[
w = 1 w \quad M f) \quad ; \quad (16)
\]
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\[
\sum_{m=1}^{2} q(m) \Xi_{mi} \Xi_{n+m} \quad \text{v} \quad (17)
\]

\[
\begin{align*}
\text{Um} & = 2 \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) + \sum_{n=m}^{2} \text{v} ; (18) \\
\text{Wn} & = 2 \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) + \sum_{n=m}^{2} \text{v} ; (19)
\end{align*}
\]

\[
\begin{align*}
0 & = 2 \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) \Xi_{n+m} + \sum_{n=m}^{2} \text{v} ; (20)
\end{align*}
\]

\[
\begin{align*}
\text{C} & = 2 \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) \Xi_{n+m} \Xi_{n+m} + \sum_{n=m}^{2} \text{v} ; (21)
\end{align*}
\]

\[
\begin{align*}
\text{E} & = 2 \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) \Xi_{n+m} \Xi_{n+m} \Xi_{n+m} + \sum_{n=m}^{2} \text{v} ; (22)
\end{align*}
\]

\[
\begin{align*}
\text{KL} & = m \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) \Xi_{n+m} \Xi_{n+m} \Xi_{n+m} + \sum_{n=m}^{2} \text{v} ; (23)
\end{align*}
\]

3.1 Parametric form and update equations for \( q(f) \) Following Miskin [1] and Miskin and MacKay [2], we find the form of the ensemble distribution \( q(f) \), by taking the variational derivative of \( C \) with respect to \( q(f) \),

\[
\begin{align*}
\log p(f) & = 2 \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) \Xi_{n+m} \Xi_{n+m} \Xi_{n+m} + \sum_{n=m}^{2} \text{v} ; (24)
\end{align*}
\]

where \( m \) are such that \( \sum_{m=1}^{2} \sum_{n=m}^{2} q(m) \Xi_{n+m} \Xi_{n+m} \Xi_{n+m} + \sum_{n=m}^{2} \text{v} = 1 \), and the notation \( f_{m} \) corresponds to the vector \( f \) with its kentry removed. Simplified expressions for some of these expectations and formulas are in Sect. 5 below. But, in order to provide some examples, the next section includes derivations of the parametric forms of two of the ensemble distributions (\( q(f) \) and \( q() \)) as well as the update equations for their respective parameters (\( u_{k}; w_{k} \) and \( u; w \) ) listed above.

\[
\begin{align*}
q(fk) & = m \exp(hlog(\text{K})/\text{KL} + \text{hlog}(\text{K})) ; (25)
\end{align*}
\]

\[
\begin{align*}
q() & = m \exp(hlog(\text{K})/\text{KL} + \text{hlog}(\text{K})) ; (26)
\end{align*}
\]
We can now expand the summand, \( q(l_i \oplus m \oplus X \oplus m_{\text{link}} \oplus k_{\oplus k}) \):

\[
\begin{align*}
&= 2 + 2 \sum_{q_{\oplus k}} X \cdot m_{\text{mix}} f_k \\
&= B^k + C^k f_k (\text{for } k = 0 \text{ and } q_k(f_k) = N_R(u_k ; w_k), \text{where } u_k ; w_k)
\end{align*}
\]

with respect to \( q() \) we obtain

\[
\frac{dC_{\text{KL, KL}}}{dq()} = \log q() \log p()
\]

\[
\sum_{n=1}^{\infty} (\sum_{k=1}^{\infty} (B^k + C^k f_k)^k)_i \quad (27)
\]

\[
k(f_k) / p_k(f_k) \exp
\]

\[
\cdot \quad (28)
\]
Substituting $p_k(f_k)$ into this expression, we finally arrive at

\[ q = \prod_{i=1}^{X} \frac{2^{N_{ki}}}{N_{ki}} \exp \left( \sum_{i=1}^{N_{ki}} q_i \left( \sum_{k=1}^{2} f_k \frac{z^k}{k!} \right) / \exp \left( B_k f_2 + C_k f_1 f_2 \right) \right) \]

where the expression on the right is otherwise.

After completing the squares, this last equation leads to the square as defined in Eqs. 132.

\[ q \]

P = 0
which, when set to zero, leads to

\[ \frac{q(\gamma)}{p(\gamma)} \exp \left( 2 \frac{q(\gamma)}{2} \right) \]
\[
\begin{align*}
q() / p() & \exp \\
+ & \cdots \\
= & G \\
+ & H \\
+ & \cdots \\
\text{and} \\
\text{substituting} \\
\text{into} \\
\text{the expression} \\
\text{for} \\
q() \\
\end{align*}
\]
\[
q_N \left( \frac{G_{i2}}{2} + H_i \right) \text{: (32)}
\]

As before, we expand \( p \hat{=} 0 \hat{=} 0 \) as

\[
\sum_{\text{term}} M_{i} \times m_{fs} \times m_{fs} \times m_{link} \times f_{k1} \times A_{2+q} \hat{=} \phi_{i2}
\]

\((^\text{Mfl})^2 q_{i2}\)

\[
\sum_{u;w} \hat{=} \sum_{u;w} \text{ where}
\]

Then, as before, we expand \( *0 \hat{=} \hat{=} \) (\( 's6\text{-}x\times M_{i} \times m_{fs} \times m_{fs} \times m_{link} \times f_{k1} \times A_{2+q} \hat{=} \phi_{i2}\))

\[
X_{TmM_i} \times X_f
\]
4 Cost function $C_{KL}$ in Algorithm 1

To derive the expression for the cost function $C_{KL}$ in Algorithm 1 of [3] we follow Miskin [1] and Miskin and MacKay [2] to approximate some of the terms in Eq. 11 of [3] with their upper bound. This allows us to re-write this expression as

$$C_{KL} = C(f_{KL}) + C(q_{KL}) + C(q_{KL}) < \log p(D_j) >$$

with each term as given below.

$$C(f_{KL}) = \sum_{n=1}^{N} \sum_{m=2}^{M} \log_2 \frac{w_m}{2w_1} m^{2} \quad q \quad \left( m \right)$$

$$C(q_{KL}) = \sum_{i=1}^{M} \sum_{m=2}^{M} \log \frac{m^{2}}{2w_1} \quad q \quad \left( m \right)$$

$$C(q_{KL}) = \sum_{k=1}^{K} \log \frac{w_k}{2w_1} \quad q \quad \left( k \right)$$
Simplified expressions for Eqs. 13-23:

\[ \log p_i = \log \frac{\sum_{k=1}^{K} m_{ik}^2}{\sum_{k=1}^{K} m_{ik}^2 + \log(\text{diag}(\text{cov}(M)))} \]

where the notation \( \text{diag}(X) \) corresponds to a vector whose entries are the diagonal entries of matrix \( X \).
$X^T$ corresponds to the vector $X$ with its $k$ entry removed, $M_{ref}$ refers to the matrix formed by removing the $k$ column of $M$, and $M_{is}$ the matrix formed by removing its $k$ column.
References