Addendum to: Intermediated Trade

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Abstract This addendum provides the proofs of Propositions 4-6 in Section 6 of our main paper.
Through out th is ad den du m, we restric t ou rse lve s to equilibria in w hic h th e intermedia tion le ve l and the agents' e xp e cted lifetime utilities immed iately j ump to th eir new steady y state valu es after W- and M- integra tion. Like in our main pap e r, on e c an sh ow that su ch equilibria a lways exist. For exp ositio n al pu rp os es , we also ignore is sue s related to the en dogen ous sep aration of matched f arme r-r-trader p airs un de r M -inte gr ation . Details are available up on reques t.

1 Large Northern Traders

1.1 Assumptions

Compared to the mo d el d esc ribed in S ection 5 of our main pap er, we as sume th at each Northern trad er ac tive in the S ou th now b e lon gs to one of T trad in g c ompanie s. Th e nu mb er of North ern trad er in g c om panies T is exoge nous ly given , bu t each comp any c an c o m plet el y ad jus t th e me as ure u x > 0 of trade rs th at it emp loys in th e S outh. For simplicit y, we restric t t ourselv es to symme tric equilibria in w hic h all Northern trading companie s h ave th e s a me siz e at all p oin ts in time. Th e com mon size of Northern trading c om panie s as well as the m easu re re of Sou them traders ac tive in the South are en dogen ous ly de ter m ined throu gh th e f oll ow ing f re e e ntry con d itio ns :

\[ V_T = V_U = 0, \quad (1) \]

The matching b etwe en each in divid ual memb ers of th e trading company and S ou thern farmers is as de scrib ed in S ection 5 of our main pap er. Bargaining, h owever, n ow procee ds un der the common knowled ge th at if a farmer ref use s to trad e with a given me mb er of a trading company, othe r me mb ers of th at trading c om pany will s top intermed iating on her b e half u ntil she has b een match ed with a Sou thern trade r r or an oth er North ern trading c om pany. Hen ce, th e Nas h bargai ning con su mption levels of a S outhe rn farmer -No rth e r trading com panies now solves

\[
\begin{align*}
\min_{C_T, S_T} & \quad p C_T + S_T + (1-a) (1 - I) \\
\text{s.t.} & \quad V_U \leq V \leq U_F, \quad \forall \text{Northern and Southern farmers}.
\end{align*}
\]

We are thus a ssum ing th at matching has a c lea ns in g e ffect on a farmer sp ast behavio r. Thi s con e a pu shed f armer has be en matched with an oth er trader, eithe r from th e North o r th e South, he c an no l o n ger be rec ognized by the Northern trading c ompan y th at ha d pr e vio us ly o st n ized h im. Wh ile this as sum ptio n is ad mitted ly ad -h oc, it con sid er a bly s impli e st h e analy sis below. Si nce newly matched f armers c an no longe r be pun ishe d, trad ers margins are in dep end ent of f armers h istory.

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1
1. Prediction

In the present environment, the state values of the interdiom level
\[ V_{VM}^+ + V_{VM}^- + V_{VT} (p > N_{VmT}) \]

For \( V_{VM^{TN}} \)
\[ V_{VM^{TN}} = \frac{U_T}{U_T + V_M} (p \leq N_{VMT}, (7)) \]
\[ L \]

For \( V_{VM^{TS}} \)
\[ V_{VM^{TS}} = \frac{U_T}{U_T + V_M} (p \leq N_{VMT}, (8)) \]
\[ L \]

For \( V_{LM^{TS}} \)
\[ V_{LM^{TS}} + V_U V_{MT} . (9) \]
\[ L \]

Where \( U_T / (U_T + U_L + L) \) is the share of unmatched Northern traders active in the Southern land in the presence of large Northern traders; and all other variables are defined in the same way as in your main...
Using the previous expressions and equation (1), we can establish the following proposition.

Proposition 4 The equilibrium under M-intensity
Proof. Equations (6)-(9) imply
\[ L, (10) \]

Similarly, equations (2)-(4) imply
\[ L, (11) \]
\[ V_{M} M_{F} = U_{L} = 1 + S \]

\[ V_{M} (1 + r + r) + F_{N} + F \]

\[ V_{M} L_{F} + F + F_{S} \]
From Nash bargain in $g$, we know that

$$V_{UTS} = V_{VMN} V_{MF} = V_{VMT} V_{MN}$$

Combining equations (10)-(15), we get, after simple rearrangements,

$$V_{UTN}$$

The two previous equations immediately imply $N > S$. To see this, suppose that $N < S$.

Since $N > S$, the two previous equations therefore imply
which implies \( N > S \) and contradicts \( N < S \). Since \( N > S \) and \( N < r \), the same logic as in our main perspective applies to our main integration. The share of our
By equations (1), rearrange equation (16) as

\[ N = L \left( 1 + \frac{r}{L} \right) \left( \frac{F + r + \frac{T}{L} + r + \frac{T}{1 + L}}{\frac{T}{L} + r + \frac{T}{L}} \right) \]

or

\[ v_{pw} = \frac{N}{v_{pw}} = L \left( 1 + \frac{r}{L} \right) \]
To conclude, let us define

Combining equations (17) and (18), we finally get

Equations (16) and (17) implicitly determine the Northern traders margin, and the intermediation level, \( r + \). After simple algebra, we can

\[ L + F = r + + 3 \]
which are just the counterparts of equations (36) and (37) in our main paper. Thus the traders, North ern traders and the intermediation level, $LL$, are identical to those that would prevail in an equilibrium with infinite similarly small North ern traders with primitive bargaining. Given equations (2)–(9), the expected lifetime utilities of all agents are identical as we ll. QED

2 Endogenous Number of Farmers, Exogenous Number of Traders

2.1 Assumptions
Pre ferrences, technology, matching, and bargainin g are as de scribed in Sec tion 2 of our main paper. Compared to our main paper, we assu me that an exogene ous measure $N$ of the island inhabitants are traders, and for simplicity, that these traders are connected to Walras ian mark ets at zero cost, $z = 0$. Conversely, we assume that there is a large pool of potential farmers who can decide at any point in time to become active or inactive. As in Section 2 of our main paper, active farmers get zero utility per period when unmatched, but stand to obtain some remuneration when matched with a trader. By contrast, in active farmers are now involved in a non-market activity that generates a constant expected lifetime utility, $V_{FT}$, e.g., subsistence agriculture. We assume that the pool of potential traders is large enough to ensure that the number of farmers operating on the island, $N$, is not constrained by population size and that some agents are always involved in subsistence agriculture. Hence, in equilibrium, $N$ is endogenously pinned down by the indifference condition:

$$V_F = V_{FT}. \quad (19)$$

The rest of the model is as described in our main paper. In particular, we assume that $\omega$ when analyzing the consequences of $M$-integration.

2.2 Predictions
For future reference, let us first describe the Bellman equations characterizing the expected lifetime utilities of the different agents in our economy. Under autarky and $W$-integration, $s$ in $c$ce $= 0$, we simply have

$$V_{MF(T)} = V_{MF}\left(T\right) - \omega(\rho) + V_{MF}, \quad (21)$$

$$\begin{align*}
V_F &= 1 - V_{MF}\left(T\right) + V_{MF}, \quad (22) \\
V_{UF} &= (1 - \omega(\rho)) + V_{MF}, \quad (23)
\end{align*}$$
Under M-integration, the Bellman equations depend on the share of unmatched Northern traders active in the South:

\[
\frac{S_W}{V_M} + \frac{(1 - \alpha) V_M F_S}{V_U F}, \quad (24)
\]

\[
\begin{align*}
V_M F_N & = F, \quad V = N, \quad (27) \\
V_M U_N & = (p_W M_{TN} + V_U T_N) V_M F_N, \quad (26)
\end{align*}
\]

Combining equations (31)-(32), we obtain

\[
\begin{align*}
V_M T & = V_M F, \\
V_M T & = V_M T
\end{align*}
\]

where

\[
\begin{align*}
V_M T & = V_M T \\
V_M & = V_M T
\end{align*}
\]

\[
\begin{align*}
V_M (p_T) + V_M & = T \\
V_M (p_S) + V_M & = S
\end{align*}
\]

\[
\begin{align*}
W & = M U_T N \\
V_M T & = (29) M T S \quad (30)
\end{align*}
\]
is the intermediation level in the Southern island under M-integration and all other variables are defined in the same way as in our main paper. Using the previous expressions and equation (19), we can establish the following proposition.

Proposition 5 Suppose that there is an endogenous number of farmers and an exogenous number of traders. Then W-integration worsens the farmers' terms of trade, increases the traders' terms of trade, and makes all agents (weakly) better off. By contrast, M-integration always creates winners and losers and may decrease aggregate welfare.

Proof. We decompose our proof into three parts. First, we compute the traders' margins and the level of intermediation under autarky. Second, we analyze the consequences of W-integration. Third, we analyze the consequences of M-integration.

Traders' margins and intermediation level. Since farmers play the same role as traders in our main paper, we can use the same logic as in Appendix A to compute the traders' margin, and the intermediation level, under autarky. By equations (20)-(23), we know that

\[ V_{UT} = v(p)r \quad (31) \]
\[ V_{UF} = (1 - V_{MF}) v(p)r + f(\ ) \quad (32) \]

We also know that Nash bargaining implies

\[ V_{UT} T = V_{MF} V_{UF} V_{UF} \quad (33) \]

\[ = (1 - V_{MF}) V_{UF} \quad (34) \]

\[ = \frac{1}{5} [(\ ) v(p)] f \]
Equations (34) and (35) imply

\[ V_{MF} = r V_F F(1) \, . \] (35)

Equations (34) and (35) imply

\[ r V_F F(1) V_{MF} = V_F^{(p)} \] (36)
When all get $v(p) = r^+ + TF(1 \times 0) \times (37)$. (36, 37) are counter parts of equation (17) and (20) in our
For future reference, note that equations (36) and (37) further imply:
\[
\begin{align*}
p_r &= \alpha_{SW}.
\end{align*}
\]
Accordingly, all Southern farmers involved in market activities will immediately specialize in coffee production, which will raise the indirect utility all matched farmer-trader pairs from \(v(p)\) to \(v_s + \mu V F T M\), as we did in the proof of Proposition 4.

Consequences of \(M\)-integration. Let us now turn to the consequences of \(M\)-integration. Combining equation (19) with our Bellman equations, (24)-(30), and invoking Nash bargaining, we can compute the Southern and Northern traders' share, and the intermediation level, \(M\), as we did in the proof of Proposition 4. Simple algebra adds to:
\[
\begin{align*}
\partial p_r &\leq \partial p_c,
\end{align*}
\]
which will, in turn, worsen the terms of trade and improve the traders' terms of trade. Using the same logic as in our main paper, it is then easy to check that all agents are better off under \(W\)-integration.
Note that with strict inequality if \( > 0 \). Using equations (19), (24), (25), (26), (29) and (30), we can also express the expected lifetime utilities of the different
\[
V = (p + V_F)^{r + (p \cdot M)} + (44) \cdot V (p \cdot M) \cdot (45) + (46)
\]

agents as follows:

Equations (42), (44) and (45), together with Nash bargaining further imply

By equations (19), (21), and (23), we know that
Equation (46) implies that M-integration always creates winners and losers: if \( V_{UTS} \) > 0 and vice versa. To establish that M-integration may also decrease aggregate welfare, we note that at any date before M-integration and after W-integration, aggregate welfare is given by

\[
V_{UF}(t) + V_{UT}(t) + V_{UTS}(t) + [N_{FU}(t)] V_{MF}(t) + V_{MT}(t) = v(p \cdot W_r + \cdot + V_{UTS}).
\]

Thus aggregate welfare can be rearranged as

\[
V_{UF}(t) + V_{UT}(t) + [N_{FU}(t)] V_{MF}(t) + V_{MT}(t) = [N_{FU}(t)] r + v(p \cdot W_r + \cdot + V_{UTS}).
\]

Before M-integration, the exact same logic implies that changes in social welfare caused by M-integration, \( W \), must exceed changes in the expected lifetime utility of unmatched traders, \( V \). By equations (39) and (44), we know that the expected lifetime utility of unmatched Southern traders after M-integration is given by

\[
V_{U} = \frac{r + V_{TF}}{V_{F}} \quad \text{for M-integration}, \quad \text{the exact same logic implies}
\]

\[
V_{U_{TS}} + \frac{r + V_{TF}}{V_{F}} + \frac{T}{7} \left( p \cdot W_r + \cdot + V_{UTS} \right) = v(p \cdot W_r + \cdot + V_{UTS}).
\]
the two previous equations implies $V_{UT}$ and such that the expected lifetime utility $V_{UT}$.

Since $M$, and Northern market institution $0$, and in turn, $W = 0$. To conclude note that if a positive measure of Northern traders are active in the South, then we necessarily have $V_{UT} < 0$, and in turn, $W < 0$. Since one can always find Northern productivity levels, 1 = $a$ of unmatch ed Northern traders is lower than their expected lifetime utility in the South in the absence of Northern traders, $V_{UT} = 0$, $M$-integration may decrease aggregate welfare. QED

3 Occupational Choices

3.1 Assumptions

Compared to Section 2 in our main paper, we now assume that the land is inhabited by a measure $L$ of agents who, at any point in time, can either become farmers or traders. For simplicity, we also assume that traders can be connected to Walrasian markets at zero cost, $\gamma = 0$, as in our previous extension. The rest of our model is unchanged. In terms of equilibrium conditions, the key difference between the model described in our main paper and the present one is that the expected lifetime utility of unmatched farmers and traders must now satisfy:

$$V_{UT} = V_{UF}, \text{ if } N_T > 0.$$  
$$V_{UT} > 0,$$  

This is the counterpart of the free entry condition, equation (10), in our original model. In any non-degenerate equilibrium with both types of agents $b$ and $e$ active, the previous conditions, of course, imply that agents must be indifferent between $b$ and $e$: $V_{UT} > 0, V_{UT} > 0, \gamma > 0, V_{UT} > 0$. (47)

The rest of the model is as described in our main paper. In particular, we assume that $> \gamma$ when analyzing the consequence of $M$-integration.

3.2 Predictions

Like in the previous extension, the Bellman equations characterizing the expected utilities of the different agents in our economy under autarky and $W$-integration are given by equations (20)-(23) and the expected lifetime utilities under $M$-integration are given by equations (24)-(30). Using the previous equation and $d$-equation (47), we can establish the following property...
Proposition 6 Suppose that all agents can become farmers or traders at any point in time. Then, $W$-integration does not affect the farmers and traders terms of trade and makes all agents better off. By contrast, $M$-integration may create winners and losers and decrease aggregate welfare.

Proof. We again decompose our proof into three parts. First, we compute the traders margins and the level of intermediation under autarky. Second, we analyze the consequences of $W$-integration.

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By Nash Bargaining, we know that

\[ V_{UF} = V_{UT} = V_{MF} = V_{UF} = 1 \]  \[ (48) \]

Let us now turn to the expected lifetime utilities of the different agents under autarky. Equations (20) and (21) imply

\[ V_{UT} = (1) V_{MF} \]

\[ = (1) (p) + F(\delta), (49) \]

\[ = (1) [r + F(\delta)] V_{MF} + F(\delta), (50) \]

Combining the two previous expressions we obtain

\[ V_{MF} = (1) \]

\[ V_{UT} = (1) \]

Similarly, equations (22) and (23) imply

\[ r V_{UT} = r V_{MF} \]
\[ \tau(\cdot) v(p) r + \tau(\cdot), (51) \quad = [r + \tau(\cdot)] v(p) r + \tau(\cdot). (52) \]
or future, note that using equation (48), the previous expression can also be written as:

\[ r + (1) \]

\[ \text{hr} + F + T_1 \]

\[ 1 \cdot (54) \]
Consequences of W-integration. We are now ready to discuss the consequences of W-integration. Like in our main paper, upon integration, the relative price of coffee will jump to p = a C s. Accordingly, all southern farmers will immediately specialize in coffee production, which will raise the direct utility of farmer-trader pairs from v (p) to v W-W-integration, therefore leaving...
the level of interaction unchanged, and by equation (53).
By equation (48),
the traders' margin, \( w = \). Thus W-integration does not affect (\( \pi \) is farmers' and traders' terms of trade). In any event, equation (49) immediately imply that W-integration makes all agents better off.

Consequences of M-integration. Let us now turn to the consequences of M-integration. To simplify, we restrict ourselves to a situation in which Northern productivity levels, \( 1 = a_C \) and \( 1 = a_S \), and Northern market institutions and are such that the expected life time utility \( V_T = V(w) \) before and after M-integration is exactly equal to the expected life time utility \( V_U W \) of a Southern trader W-integration. Thus the share of unmatched Northern traders is \( 1 \). We will verify at the end of our proof that there indeed exist parameter values such that this is optimal for all Southern agents to become farmers after M-integration.

Let us start by computing the Northern traders' margins, \( N \), and the intermediation level, \( M \), as we did in the proof of Proposition 4. Combining equations (24), (26), (27), and (28), and invoking Nash bargaining in g, we obtain after simple algebra

\[
W(t) = u^M_{UT} \frac{r f G^M}{r G^M + 1} V_T(p) \quad (55)
\]

and

\[
W(t) = u^M_{UT} \frac{r f G^M}{r G^M + 1} V_T(p) \quad (56)
\]

Note also that if \( V = V_U T \) is decreasing in \( V \), then \( W = V_U T \) and increasing in \( v \) and \( p \). Thus under our assumptions in at the level of intermediation will increase in the South under M-integration in the proposed equilibrium.

Compared to the proof of Proposition 5, it is convenient to show first that M-integration may decrease aggregate welfare, and that at those circumstances, it may also lead to distributional conflicts. At any date before M-integration and after W-integration, aggregate welfare is given by

\[
V_T^{(t)} + V_M^{(t)} = M^{(t)} + V_M^{(t)}
\]
By equations (47), (21), and (23), we know that

$V_u F(t) + [N_F(t) + V_M(t) + V_M F(t) + V_M T(t)] = v(p w_\tau + r + (t + V_U F(t) + V_U T(t) + V_U , F)

Thus aggregate welfare can be rearranged as

$W(t) = V_U F(t) u_F(t) + (p w_\tau(t) + 2[N_F(t)] r + 10) r +$
is not affected by M-integration and \( u_F(t) \) and \( u_F(U_F(t)) \) are predetermined at date \( t \), the previous expression implies that if all Southern age nts become farmers after M-integration, which, as mentioned earlier, we will check at the end of this proof, changes in social we are caused by M-integration, \( W \). By equations (20) and (21), we know that the changes in the expected life time utility of unmatch ed farmers, \( V \), must reflect changes in the expected life time utility of unmatch ed farmers caused by M-integration.

**Lemma 1.** Suppose that only Northern traders are active in the South, then \( dV_{U_F}; u_T, d \ln u_T = 0 \) if and only if \( d \ln m(t) \). Proof. We proceed in two steps. Step 1: If only Northern traders are active in the South, then the expected lifetime utility of unmatch ed farmers satisfy \( V_{U_F} = 1 \).

Like in our main paper, we will now show that \( V_{U_F} = 1 \). (58)

Combining this expression with equation (55), we obtain

\[ r = V_{U_F} = F = M = 1. \]

After simple algebra, equations (56) and (59) imply

\[ r V_{U_F} = \frac{N D}{M + v_{pw}}. \]

which can be rearranged as

\[ \frac{r V_{U_F}}{1} = \frac{N D}{M + v_{pw}}. \]
Equation (58) directly derives from the previous expression and equation (56).
Step 2: If only Northeast and South are active in the South, then the expected longevity of un...
with \( (r+1)! \left( \frac{1}{N} \right) + 1 \) we get
\[
\frac{dV}{F u T} = V \frac{T m}{m} \left( r + \frac{1}{2} \right) - 1. \quad (61)
\]

By differentiating equation (58), we have
\[
\frac{dV}{F u T} = M \frac{V}{m} \left( r + \frac{1}{2} \right) - 1. \quad (61)
\]

South, a hypothetical increase in the bar gain in power of...
Northern trades decrease aggregate welfare if $\gamma > 0$.

A direct corollary of Lemma 1 is that if M-integration leads to the exit of all untaxed Southern trades, as proposed in this equilibrium.
then it must also decrease aggregate welfare in the South whenever.

> > " "

We now demonstrate that there exist parameter values such that only Northern traders are
active under M-intergration.

To do so we build on the following lemma.

Lemma 2

Suppose that only Northern traders are active in the South. Then there exist $t > 0$ and $s > 0$ such that
and 2 ( ; + )

where

$\text{VU}_T$ is the expected lifetime utility that a Southern would get if he were to become a trader after M-integration.

Proof.

By eq...
known that By directed direction (56), it is easy to check that the stack is increasing in $r$. Equation (60) directly derives from equations (61) and (62). By Step 2, if Northern traders are active in the South, then $V=d_0$ if and only if \ldots$.
Equations (63) and (65) imply
\[ r + + F = M \]
\[ r + + T = V U T S \]
\[ M \]
\[ V U F \]
\[ M T S \]

Let us now express as a function of \( r + + T \). By equations (22) and (23), we know that \( V \) Similarly, by equations (20) and (21), we know that \( V = M V \) (68)
We also know that Nash bargaining implies \( V = M V M V U F \) (69)

\[ V U F = M T U T = V M F U F V \]
\[ (r + ) M \]
\[ (1 \) \]
\[ S V S S V \]

\[ M W \]
\[ u \]
\[ p \]
\[ i \]
\[ l \]
\[ y \]
\[ n \]
\[ g \]
\[ e \]
\[ q \]
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\[ m \]
\[ p l \]
Together with equation (67), this implies
\[
V_U V_F^{NSSTS} = \frac{M}{F} (1) M V M V_U W T T F. (71)
\]

By equation (63), we know that
\[
(1F N D) r + + N = + + \frac{M}{F} (1) M V M V_U W T T F. (72)
\]
Combining this expression with equation (72), we obtain

\[ r + r + V + T = T ! + v 1 ! . \]
This implies
\[ \frac{d}{dV_U} \ln V_T = \frac{d}{dV} \ln V_F < 0 \]
and
\[ \frac{d}{dV_U} \ln V_M = \frac{d}{dV} \ln V_M > 0. \]

It is easy to check that
\[ \frac{dV_U}{dV} = v = 0, \text{ we therefore have } \]
\[ \frac{d}{dV_U} \ln V_U = v > 0. \]

Lemma 2 derives from the previous inequality and the fact that only Northern traders are active in the Southern under M-integration if is also close enough to . Combining this observation with Lemma 1, we have demonstrated the existence of an equilibrium such that aggregate welfare is strictly lower under M-integration than under W-integration.

To conclude our proof we now demonstrate that M-integration may also create winners and losers. The argument is similar to the proof of Lemma 2. By equation (29), we know that
\[ \frac{d}{dV} \ln V_T = \frac{d}{dV} \ln V_F. \]

Differentiating we therefore have
\[ \frac{d}{dV_U} \ln V_M = \frac{d}{dV} \ln V_M = \frac{d}{dV} \ln V_U > 0. \]

It is easy to check that
\[ \frac{d}{dV_U} \ln V_M > 0, \frac{d}{dV} \ln V_M > 0. \]

The fact that M-integration may create winners and losers directly follows from this observation. QED