Long-term relationships: Static gains and Dynamic inefficiencies

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Abstract

This paper formalizes the idea that firms can (partly) overcome the static costs associated with low contractibility by engaging in long-term relationships, but that the rigidity of these relationships may lead to strong dynamic inefficiencies. Specifically, we consider the repeated interaction between a final good producer and an intermediate input supplier. A pair producer/supplier can be a good match or a bad match and the nature of the match does not change over time. As a consequence, learning that a match is good increases the value of the relationship and creates the potential for cooperation in general equilibrium. We then allow for innovation on the supplier side and show (i) that innovations need to be larger for a new supplier with superior technology to break up existing relationships than in either a set-up where the input is contractible or when we preclude cooperation in long-term relationships, and (ii) that the rate of innovation is lower than in the contractible case, and may be lower than when cooperation is precluded in the noncontractible setting. We then show that, when innovation is exogenous, cooperation in long-term relationships may overcome most of the welfare loss associated with non-contractibility, but that, once innovation is endogenized, welfare is much lower in the noncontractible case than in the contractible case, and can even be lower if cooperation occurs in long-term relationships than if it does not. However, we show that far from the technological frontier, the establishment of long-term relationships can turn out to be a good substitute to high contractibility. An analysis of patents data in the US provide motivational evidence to our work.

JEL.

Keywords: contractibility, long-term relationships, innovation.

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1 Introduction

Do contractual frictions still matter when firms engage in repeated interactions? Although recent work in the growth literature has emphasized the central role that institutional features like contractibility play in shaping income and growth differences across countries, at the same time, there is widespread evidence that firms engage in implicit contracting that could substitute for formal contracting (for instance [Johnson et al., 2002] shows in a survey across Eastern European countries that after the first year of a relationship the extent to which parties trust the court has a very small impact on trust towards one another). At first glance, cooperation in long-term relationships could then overcome the inefficiencies of a poor contractual environment. This paper shows that implicit contracting is in fact a very poor substitute for formal contracts once dynamic aspects are introduced. Indeed, cooperation in long-term relationships comes at a price: stickiness in relationships; firms may prefer to stay with a known partner with whom they are used to cooperate rather than risking the switch to a new partner with access to a more productive technology. In fact, [Johnson et al., 2002] has shown that the belief in courts’ efficiency has a strong impact on the incentive for firms to try out new suppliers. Rigidities in relationships, in return, reduces the incentive for entrants to innovate, up to a point that the economy might even be worse off when cooperation arises in long-term relationships than when it does not. Hence, the possibility of developing cooperation in long-term relationships turns contractibility issues from a static problem of inefficient allocation of resources into a dynamic problem of inefficient development of technologies. The idea that long-term relationships can overcome institutional weaknesses but at the cost of adding “rigidities” in the economy, is not new in the literature ([Johnson et al., 2002] evokes this possibility), but, to our knowledge, it has not been formally analyzed yet. In this paper, we provide a first attempt at such an analysis, and we investigate some of the macroeconomic consequences, focusing in particular on growth and innovation. However, rigidities in relationships could potentially affect the economy in other dimensions, for instance the ability to respond to macroeconomic shocks as well.

More specifically, we consider an industry where production requires that a final good producer and an intermediate input supplier work together, but where the supplier is the only one making an investment. We contrast the case where the provision of the input is contractible with the case where it is not. In a non-repeated framework, noncontractibility typically creates an ex post hold-up situation leading to underinvestment by the supplier as in [Grossman and Hart, 1986]. The match between the producer and the supplier can be “good” or “bad”, where good matches are characterized by a higher productivity. The nature of the match is unknown to both parties before they start working together and it does not change over time. Therefore, if a match turns out to be good, the value of the relationship in the following period is higher than the expected value of a new relationship. In the noncontractible case, this
difference in values allows for an equilibrium where in a good match the supplier invests more than he would in a one shot interaction, even though producers are able to switch suppliers at will and suppliers do not coordinate amongst themselves to punish producers who switch suppliers.

Having introduced the possibility of repeated interaction partly overcoming contractual incompleteness, we go on to demonstrate the possible inefficiencies of established relationships. We allow a new supplier to enter the market with a better technology. Producers already engaged in a long-term relationship face a trade-off: switching to the new supplier allows them to have access to a more productive technology, but at the risk of entering into a bad match. Entering into a bad match always yields lower production because of the inherent lower productivity of the relationship; but, when the input is noncontractible and when suppliers cooperate in good matches, bad matches are also characterized by more severe under-investment than good matches. Hence, bad matches are even worse relative to good matches, this worse bad match effect is the principal force behind our result that cooperation in a weak contractible setting magnifies rigidities in relationships. First, we show that, in order for an innovator to capture a large share of the market, innovations have to be more radical when in the noncontractible case with cooperation (which we label as the “cooperative case”) than in the contractible case or in the noncontractible case when cooperation is precluded in long-term relationships (which we label as the “Nash case”). As a consequence, the model predicts more technological differences across firms in countries with poor contractibility institutions (if cooperation arises) than in countries with strong institutions. Then, we show that innovation and even welfare can be lower in the cooperative case than in the Nash case. We provide a numerical example where the establishment of cooperation in long-term relationships allows to overcome most of the inefficiencies of poor contractibility when innovation is exogenous, but is actually detrimental to welfare when innovation is endogenous.

In an extensions section, we derive additional predictions by modifying the initial framework in different ways. First, we develop a quick extension of the model where there is a fixed technological frontier that all suppliers can freely imitate on at the beginning of the period, in this case cooperation in long-term relationships turns out to be a good substitute to good contractibility institutions, not only because most of the growth occurs through imitation so that the impact of innovation is reduced but also because imitation itself reinforces cooperation. Second, we show how with a small modification of the strategies played in the cooperative case in the original set-up, the cooperative case can easily feature multiple equilibria: some with high levels of cooperation and reduced growth and some with low level of cooperation and faster growth, and how innovation may tend to happen through waves in the cooperative case. Third, we show that cooperation in long-term relationships may incentivize innovators to produce scarcer but larger innovations, and fourth, that the negative effect of cooperation
on innovation can be reduced when patents are better enforced.

Our paper relates to two main topics in the literature: the possibility of building relationships under imperfect contractibility, and the impact of institutions, in particular contractibility, on macroeconomic outcomes (growth or trade). A large body of theoretical literature addresses the question of building relationships in the presence of contractual incompleteness: the repetition of the same interaction can give rise to equilibria of the Folk theorem type, where parties cooperate and provide more effort (or investment) than they would in a one-shot interaction. [Kreps, 1996] gives a good overview of the theoretical work on this topic. A prominent paper in the field is [Baker et al., 1994], the authors address the issue of formal versus implicit contracts by considering the repeated interaction between a firm and an employee. They analyze a situation where firms rewards employees through two mechanisms: a wage that depends on a contractible signal of the employee’s performance, and a noncontractible bonus that depends directly on the employee’s unverifiable performance. They demonstrate that sometimes a signal better correlated to the actual performance (which can be interpreted as a proxy for more developed institutions) can prevent the formation of more efficient implicit contracts.¹

[Macaulay, 1963] is the first paper to show that interactions between firms in most markets are repeated and that firms engage in relational contracts. [Ellickson, 1991] is the seminal reference in the law literature arguing that, in close communities, people cooperate without the help of the state, because they are engaged in repeated interactions. [Brown et al., 2004] ran experiments showing the endogenous emergence of long-term relationships in the absence of third party enforcement, where low effort was punished by the termination of the relationship. Furthermore, they showed that in successful relationships, effort was high from the very beginning. Our equilibrium shares these features. [Banerjee et al., 2000] show that in the Indian software industry, reputation of firms matter for the kind of contracts they are offered. [Allen et al., 2005], [Allen et al., 2006] and [Allen et al., 2008] show in related papers that in India and China long-term relationships provide a successful way to finance firms. [MacLeod, 2006] provides a very interesting discussion of the different mechanisms that allow the enforcement of incomplete contracts, and compares the performance of informal enforcement with formal enforcement. [Raiser et al., 2008] use World Bank Enterprise Survey (as we do) to assess the determinants of trust in transition countries.

Since [North, 1981] a large literature has emerged on the impact of institutions on growth and development. [La Porta et al., 1998] and [Acemoglu et al., 2001] are the seminal papers which demonstrate empirically that variations in the institutional foundations can have significant influence in economic outcomes. A nice theoretical treatment is given in [Acemoglu et al., 2006].

¹In [Baker et al., 2002], this model is extended to study the boundaries of the firm. Klein and Leffler (1981) show that repeated purchases by consumers may incentivize firms to provide high quality good in a noncontractible setting, but that the price needs to be higher than the competitive price. In [Klein, 1996], firms engaged in long-term relationships, cooperate within a range of price variation, but hold-up occurs outside this range.
Productivity increases both through imitation from the technological frontier and incremental innovation, where the capacity of incremental innovation depends on the ability of the manager. Far from the technological frontier, firms pursue investment-based strategies featuring long-term relationships between firms and their managers, but sacrifice selection of managers, whereas close to the frontier, relationships become shorter and the selection of good managers becomes more important. Institutions that favor the establishment of long-term relationships are appropriate far from the frontier but turn out to be a burden close to it.\(^2\) Amongst these institutional features, the extent to which one can enforce contracts is of particular interest, as it varies widely even across developed countries. [Acemoglu et al., 2007] have shown that countries with weaker legal institutions adopt inferior technologies and develop a comparative advantage in sectors where there is more substitutability across inputs.\(^3\) Although [Acemoglu and Johnson, 2005] downplays the importance of contractibility compared to property rights institutions, [Cowan and Neut, 2007] show empirically, that productivity is relatively larger in countries with good legal enforcement in sectors with a more complex intermediate structure, and, similarly, [Nunn, 2007] show that these countries develop a comparative advantage in sectors that rely more on relation-specific investments.\(^4\) [Dixit, 2004] is closely related to our paper as it analyzes the type of informal institutions that emerge when the judiciary system of a country is not very developed yet (a famous example based on repeated interactions is [Greif, 1993]'s Maghribi traders).

Starting with [Aghion and Tirole, 1996], an important body of literature analyzes the organization of R&D in an incomplete contracts framework (see [Aghion and Howitt, 1998] for an interesting analysis of these issues), in contrast, our paper ignores the issue of the exact organization of the R&D process and focuses on the impact of the organization of the production process on innovation. The literature on incomplete contracts and macroeconomics also include: [Francois et al., 2003], who study the impact of growth on contractual arrangements (some of our results point towards feedback effects where the frequency and the type of innovation affect in return the extent of cooperation between business partners); and [Caballero and Hammour, 1998] who relate incomplete contracts with macroeconomics shocks amplification. Finally, a related idea that long-term relationships between producers and suppliers can be barriers to entry was already formalized in [Aghion and Bolton, 1987], who show that when an incumbent faces entry by potential competitors with superior technology, she will sign long-term contract that reduces the risk of entry. In our set-up, however, the relationship

\(^2\)[Bonfiglioli et al., 2009] present a similar trade-off. In their model, rigid long-term contracts that favor investment by managers, can be optimal at early stage of development but as capital accumulates, the return to talent increases and flexible contracts are more appropriate.

\(^3\)[Ottaviano, 2007] studies how contract enforcement simultaneously affects the location of R&D activities and whether production and R&D are integrated or not, and how in return this last decision affects the returns from R&D.

\(^4\)In the trade literature, [Rauch, 1999] shows the importance of networks in shaping trade, especially for more differentiated products.
is of a different nature as the contract is implicit and we rule out explicit contracts that would last more than a single period.

Section 2 presents the motivational evidence. Section 3 presents the model in a static framework, and shows how long-term relationships bring static gains. Section 4 introduces innovation in this framework, and contains our main results. Section 5 extends the model in different directions and contains additional predictions. Finally section 6 concludes and suggests future research.

2 Motivational evidence

Our motivational evidence are based on the distribution of foreign patents recorded at the United States Patents Office (USPTO) between 1992 and 2006. We show that countries with better contracting institutions tend to innovate relatively more in sectors that are more contract intensive. More specifically, we run regressions of the form:

\[ Y_{c,s,t} = \beta L_c + \beta_s X_{c,s} + \delta_s C_s + \delta_t + \varepsilon_{c,s,t}, \]

where \( Y_{c,s,t} \) is a measure of the amount of innovation by firms from country \( c \), in sector \( s \), in year \( t \), \( L_c \) is a measure of the quality of contracting institutions in country \( c \), \( X_c \) is a vector of country-specific controls, \( C_s \) is a measure of contractual intensity in sector \( s \), and \( \delta_s, \delta_c \) and \( \delta_t \) are sector-specific, country-specific and year-specific fixed effects.

2.1 Data

Patents data have been widely used as a proxy for innovation. The NBER dataset, described in [Hall et al., 2001], contains all patents recorded at the USPTO from 1963 to 2006 and the number of citations made to each patent until 2006 adjusted for the age of the patent. We complete the dataset available online with the dataset on the assignees of the patents from [Lai et al., 2009], and we focus on the 2064233 patents granted after 1992. We are able to identify the country of the first assignee for 1927079 patents, dropping the 1023038 patents from US origin, patents from countries with less than 50 patents total in the period 1992 - 2006, patents from fiscal havens\(^5\) and patents attributed to the USSR, Czechoslovakia or Yugoslavia, we are left with a total of 889208 patents. Each patent is given (at least) one technological code according to the IPC classification.\(^6\) Using a concordance between the IPC code and the ISIC classification,\(^7\) we are able to match each observation with a specific sector (at the two or three digit level), and then each patent to a specific sector by randomly selecting a single sector.

\(^5\)We drop patents from Bahamas, Barbados, Bermuda, Cayman islands, Cook islands, Netherlands antilles, St. Kitts and Nevis and Virgin (British) islands.
\(^6\)There is a total of 924914 observations for only 900346 patents.
\(^7\)We first use a concordance IPC - NACErev1 and then NACErev1- ISICrev3.
when a patent falls in several sectors. To get our first measure of innovation, we aggregate the number of patents at the country - sector - year level, add 1 and take the logarithm. However, each patent does not have the same economic value: some innovations are worth much more than others, a proxy commonly used for the size of an innovation associated with a patent is the number of citations made to the patent (see for instance [Trajtenberg, 1990]). For our second measure of innovation we then aggregate the number of patents at the country - sector - year level with the number of citations made to these patents, add 1 and take the logarithm. For our third measure, we replace the number of citation as measure of the size of one innovation by the logarithm of 1 plus the number of citations. For our second and third measures, we restrict attention to patent granted before 2003 ([Hall et al., 2001] advise to ignore the last three years of data when using citations).

The measures of contracting institutions that we use are from the World Bank ([Bank, 2004]) and have been used in several papers including [Acemoglu and Johnson, 2005]. Following the methodology of [Djankov et al., 2003], the World Bank collected data on the procedures involved when a firm tries to collect a commercial debt contract worth 50 percent of the country’s annual income per capita from a reluctant buyer. They computed two measures: the total number of procedures and an index (on a scale from 1 to 10) of the degree of legal formalism involved. The underlying assumption is that contracts are easier (and less expensive) to enforce when the number of procedures and the degree of legal formalism is limited. The control variables that we use are GDP per capita in 1992 (from the World Bank) and the measure of protection against expropriation by government averaged over 1985 - 1995 from Political Risk Services (this measure is used for instance in [Knack and Keefer, 1995] and in [Acemoglu and Johnson, 2005]).

Our measure of contractual intensity is based on the latest version of the classification of goods elaborated by [Rauch, 1999]. Rauch classifies goods (at the 4 digit SITC rev2 level) depending on whether they are sold on organized exchange, reference priced in catalogs or neither. Presumably, goods sold on an organized exchange market are quite homogenous with a thick market, while goods that are not even reference priced are more differentiated and specific to a limited number of buyers. The scope for hold-up is then larger for the latter goods than for the former, and as several other studies have done (for instance [Nunn, 2007]), we can use this classification to proxy for the contractual intensity of a sector. More specifically, we build an index taking the value 0 when the good is sold on an organized exchanged market, 0.5 when it is reference priced and 1 when it is neither sold on an organized market nor reference priced. We use concordance tables to compute such a measure at the ISIC rev3 2 digits and 3 digits levels.\footnote{Rauch gives a conservative and a liberal classification, we will use both in our empirical analysis.} We use concordance tables going from SITCrev2 to SITCrev3 and then SITCrev3 to ISICrev3.
In Appendix Z, we report a table giving the distribution of patents across countries together with the measure of contracting institutions in the country, and we report a table describing the distribution of patents. Because a higher value of \( L_c \) indicates worse contracting institutions and a higher value of \( C_s \) a higher level of contractual intensity, we expect to find \( \beta < 0 \): innovation should be relatively larger in countries with good contracting institutions in sectors with a high level of contractual intensity.

### 2.2 Results

In our baseline regression we have observations for 46 countries across 33 sectors and in 15 different years. However, many observations at the country - sector - year levels are 0 (47% of the observations in our baseline case), we then run separately our regression on the intensive margin (whether there is at least one patent in a given country - sector - year) and on the extensive margin.

For the intensive margin, we adopt a logit model. The results are reported in table 1 (standard errors are clustered at the country-industry level). Columns (1) shows that in countries where the number of procedures to collect a check is higher, the probability of innovation is reduced in sectors where contractual intensity is higher (when one uses the liberal classification); the effect is significant at the 1% level. Column (3) shows that the same holds when one uses the conservative Rauch classification and column (2) shows that the formalism index has the same negative effect (significant at the 5% level when one uses the liberal classification as in column (2), it is significant at the 1% level when one uses the conservative classification). An obvious alternative explanation is that sectors with a higher contractual intensity may also be more "complex" (although a quick look at table 0.2 shows that there may not be a perfect correlation), and therefore only developed countries are susceptible to innovate in these sectors. To take into account this possibility, we include the interaction between GNI per capita in 1992 with the contractual intensity of the sector as a control variable. Even though, this control is highly endogenous, the effect of worse contractibility institutions does not disappear: it remains significant at the 10% level when one uses the liberal classification, it is significant at the 1% level with the number of procedures and the conservative classification (and at the 5% level with the formal index and the conservative classification in a non reported regression).

In column (8), we also control for the risk of expropriation; in many papers, the measure of the quality of the judicial system used mixes the quality of contracting institutions with property rights protection, here we see that the property rights variable has no explanatory power. Finally, the effect is economically significant, in table 1, the odds ratio is 0.926, so that if a country’s number of procedures moves from the 75th percentile (22, e.g. Germany) to the 25th percentile (16, e.g. Ireland), the odds ratio increases by a factor 1.14 more in the sector in the 75th percentile of contractual intensity (1, e.g. Manuf of general purpose machinery ) than in
the sector in the 25th percentile (0.71, e.g. Manuf of other electrical equipment n.e.c.).

Table 1: Intensive margin

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</table>

Note: Logit model. Table reports coefficient, standard errors clustered by country-industry in parentheses and odds ratio. All regressions include country, sector and year fixed effects. Column 4 restricts attention to countries with more than 500 patents in total. * p<0.1, ** p<0.05, *** p<0.01

We now deal with the extensive margin, that is the number of patents conditional on at least one patent being granted in the country - sector - year. We run a simple OLS regression on the sample of country - industry - year with at least one patent. The results are reported in table 2 (standard errors are clustered at the country-industry level). In all columns except column 4 the dependent variable is the logarithm of the total number of patents, in column 4, the dependent variable is the logarithm of the total number of patents plus the total number of citations made to those patents. In columns (1), (2) and (3) we find a negative and significant at 1% level effect of a higher number of procedures or more formal legal system on the number of patents in more contractual intensive sectors. Column (4) shows that the effect carries on when one takes into account the size of inventions as proxied by the number of citations made to those patents. Column (5) restrict attention to countries with at least 500 patents in total. In columns (6) and (7) we control for initial GNI per capita, the effect of the number of procedures remains significant at the 10% level when one uses the liberal classification, but the effect of the
formalism index becomes insignificant - however GNI per capita is a very endogenous variable to start with -. With the conservative classification, the effect of the number of procedures is significant at the 5% level while the effect of the index is also insignificant. Finally column (8) shows that protection against expropriation has no effect. The interpretation of the magnitude of the coefficient goes as follows: when a country’s number of procedures moves from the 75th percentile to the 25th percentile, the number of patents increase by a factor 1.12 more in the sector in the 75th percentile of contractual intensity than in the sector in the 25th percentile.

Table 2: Extensive margin

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<tr>
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<td>0.80</td>
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Note: OLS model. Table reports coefficient and standard errors clustered by country-industry in parentheses. All regressions include country, sector and year fixed effects. Column 5 restricts attention to countries with more than 500 patents in total * p<0.1, ** p<0.05, *** p<0.01

Finally, for robustness check we run a tobit model in Table 3 (with standard errors clustered at the country-industry level), where the dependent variable is the logarithm of 1 plus the total number of patents. The results are very similar (the coefficients are very close to the ones in Table 2). Note that when controlling for GNI per capita, the effect of the number of procedures is now significant at the 5% level when using the liberal classification (in unreported results, the effect of the number of procedures is significant at the 1% level and the effect of the degree of formalism is significant at the 10% level when one uses the conservative classification).
Table 3: Tobit model

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<td>Num proc</td>
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</tr>
<tr>
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<tr>
<td>Form index</td>
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<tr>
<td>*Cont int lib</td>
<td>(0.075)</td>
<td>(0.070)</td>
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<tr>
<td>Num proc</td>
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<td>-0.084***</td>
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<td>(0.019)</td>
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<tr>
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Note: Tobit model. Table reports coefficient and standard errors clustered by country-industry in parentheses. All regressions include country, sector and year fixed effects.

* p<0.1, ** p<0.05, *** p<0.01

3 Sustaining long-term relationships and contractibility

In this section we develop a model of repeated interaction between final good producers and intermediate input suppliers in general equilibrium, where some matches producer/supplier are exogenously better than others. We first show that when the provision of the input is non-contractible, the classic hold-up problem arises and suppliers have an incentive to underinvest in an one-shot interaction. We then let the game be repeated, and show that the prospect of continuing the relationship in the following period provides suppliers in a good match with an incentive to cooperate with the producer, by investing more than they would in the one-shot interaction.

3.1 Preferences and production

We consider a quasi-general equilibrium model where consumers consume only two types of final goods: a set of differentiated goods (denoted $c_i$) of measure 1, and a homogenous outside good (denoted $C_o$). Aggregate preferences are given by a representative agent with utility function:

$$U = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \left( C_{o,t} + \frac{\sigma}{\sigma - 1} \int_{0}^{1} c_j^{\frac{\sigma-1}{\sigma}} dj \right),$$

where $\rho$ denotes the discount rate of the consumers. For clarity we will drop the subscripts $t$ when this does not lead to confusion.

We assume that the productivity of the differentiated good (to be specified below) is low enough that the outside good always remain active. Each variety is produced monopolistically
by a final good producer. The demand for a variety \( j \) \( (c_j) \) and the quantity of variety \( j \) produced \( (q_j) \), can then be written as a function solely of its own price:\(^9\)

\[
q_j = c_j = p_j^{-\sigma}.
\]  

Revenues of the producer of variety \( j \) can then be expressed as \( q_j^\frac{\sigma+1}{\sigma} \).

The outside sector produces a homogenous good at constant returns to scale one for one with labor and we normalize its price to 1, such that the wage in the economy is also equal to 1. All the action in the model takes place in the production of differentiated goods. Production of each final good \( j \) in the differentiated sector requires intermediate inputs which are provided by an intermediate input supplier. Intermediate inputs can be of good or bad quality, and, only good quality inputs have value in production. Each final good producer is supplied by only one intermediate input supplier, whereas each intermediate input supplier can supply any number of final good producers without decreasing returns to scale. There is a mass 1 of intermediate input suppliers.

Every period a mass \( \delta^D \) of existing final good producers die and are replaced with new ones. Intermediate input suppliers are infinitely lived.\(^{10}\) Production of final good \( j \) is given by:

\[
q_j = (\theta_{jk}A_k)^{\frac{1}{\sigma-1}} X,
\]  

where \( \theta_{jk} \) is a match specific and verifiable permanent level of productivity, \( A_k \) is the productivity of the intermediate input supplier \( k \) and \( X \) is the quantity of intermediate inputs of good quality provided by the supplier (technically, we should refer to \((\theta_{jk}A_k)^{\frac{1}{\sigma-1}} \) as the productivity, but, throughout the paper we make the abuse of language of referring to \( \theta_{jk}A_k \) as productivity). The match specific level of productivity \( \theta_{jk} \) can take two values: \( \theta_{jk} = \theta^g \equiv 1 \) in good matches, and \( \theta_{jk} = \theta^b \equiv \theta < 1 \) in bad matches. The quality of a match is revealed to both the supplier and the producer only once they start working together and is permanent. Further, there is a probability \( b \) that a random pair producer/supplier turns out to be a bad match. Producing one unit of good quality intermediate input requires one unit of the homogenous good (bad quality inputs can be produced costlessly).

Throughout the paper we will normalize the amount of good quality inputs provided by the supplier by the productivity of the relationship \( \theta_{jk}A_k \), and denote it \( x \) (so that \( x \equiv X/ (\theta_{jk}A_k) \)). We can then express revenues as \( \theta_{jk}A_k R(x) \), where \( R(x) \) are the normalized revenues \( (R(x) \equiv x^{\frac{\sigma-1}{\sigma}}) \), and joint profits as \( \theta_{jk}A_k \Pi(x) \) where \( \Pi(x) \) are the normalized joint profits \( \Pi(x) \equiv x^{\frac{\sigma+1}{\sigma}} - x \).

\(^9\)The functional form of the utility function allows us to avoid general equilibrium effects going through the wages (thanks to the presence of the homogenous good) or the price index (as the elasticity of substitution between the varieties is equalized with the price elasticity of the CES aggregator). These general equilibrium features would complicate the analysis without changing any of our central results.

\(^{10}\)We could equally well have assumed that the intermediate good suppliers die with probability \( \delta^D \).
The central feature of this paper is the trade-off that a final good producer faces between the level of cooperation he can expect from the intermediate inputs supplier and the level of the technology of the supplier. In this section, however, we do not allow for innovation and we can normalize $A_k = 1$ for all suppliers.

3.2 Contractual incompleteness

We model contractual incompleteness in a very standard fashion (closely following [Grossman and Hart, 1986]): contractual incompleteness is the source of a classic hold-up problem. More specifically, we assume that for every producer, a different state of the world is drawn every period after the producer has chosen his supplier. To be of good quality, an input must be tailored to the specific state of the world. Further an input is specific to a particular producer and is useless to any other agent in the economy, and, after the state of the world has been revealed, a producer cannot find another supplier; therefore the set-up becomes one of bilateral monopoly. If the input is contractible, the producer and the supplier can initially sign a contract that fully specifies the characteristics of the input for each possible state of the world, whereas this is impossible if the input is noncontractible, that is, if a court cannot verify whether the input is of good or of bad quality. Noncontractibility leads to a double hold-up problem: the producer can hold-up the supplier by claiming that the input is of bad quality, while the supplier can hold-up the producer by delivering a bad quality input, and, no court would be able to check which party is wrong. Therefore, any contract specifying the amount of inputs of good quality to be provided is worthless. We consider that revenues are shared through ex-post Nash Bargaining (after the state of the world has been realized then), and we denote by $\beta \in (0, 1)$ the bargaining power of the supplier. Throughout the paper we will compare the situation where the input is fully contractible to the one where it is not.

3.3 Structure of the game and one shot version.

In every period, the timeline is then the following:

1. Final good producers die with probability $\delta^D$ and a mass $\delta^D$ of new final good producers is born.

2. Every supplier makes a take it or leave it offer of an ex-ante transfer $t$ to every producer. When the input is contractible, they can also contract on the amount of good quality input to be provided, given the nature of the match.

3. Producers choose their supplier, and the transfer $t$ from the supplier to the producer is paid.

\[11\] We make the classic assumption that revenues are observable to the parties but non verifiable, and so cannot be part of a contract.

12
4. The type of the match is revealed if the two parties are interacting for the first time (it is already known otherwise).

5. State of the world is realized.

6. Suppliers decide on how much good quality input to provide in the noncontractible case (they are bound to follow the contract in the contractible case).

7. Revenues are shared between producers and suppliers through ex post Nash Bargaining.

We restrict attention to contracts specified by the game above, so, contracts last for one period only. The fact that suppliers make take it or leave it offer to producers when deciding on the ex-ante transfer, implies that they are engaged in Bertrand competition.\footnote{The formal proof of this is in Appendix A.} As a consequence, if all suppliers are symmetric they do not capture any rent from the relationship. However, if one supplier has an advantage over the others, in the sense that the relationship with that supplier is of higher value than any other potential relationship, the supplier can capture the entire surplus of that relationship over the other potential relationships. Bertrand competition (instead of ex-ante Nash bargaining) simplifies the exposition but does not affect our results, we discuss what happens with ex-ante Nash bargaining or when producers make take-it or leave-it offers at the end of the section.

Before proceeding to describe the equilibrium that we study, we consider a one period version of the game above to demonstrate the inefficiencies that repeated interaction can overcome. Consider first the case where the input is fully contractible. Suppliers, engaged in Bertrand competition, offer the contract that guarantees the highest value to the producer, while making them still break even. Therefore, they offer to produce the amount of good quality inputs that maximize joint profits, and offer an ex-ante transfer $t$ such that the final good producer captures the entire value of the relationship. Hence normalized good quality inputs is at the first best level ($m$) given by:

$$
m \equiv \arg \max_x R(x) - x = \left( (\sigma - 1) / \sigma \right)^{1 / \sigma},$$

so that the amount of good quality inputs is $m$ in good matches and $\theta m$ in bad matches.

Now, consider the situation in which no enforceable contract can be written on the provision of the input. The normalized investment $m$ as found above is no longer in the self-interest of the supplier, as he bears the full marginal cost but only gets a share $\beta$ of the marginal benefit. Hence the supplier, once he has been chosen, would not commit to any announcement he may have made about the quantity of good quality input he would provide, instead he will choose an investment level that maximizes his ex-post profits. Ex-post profits are given by a share $\beta$.
of the revenues minus the whole cost of the investment, so normalized amount of good quality inputs provided by the supplier is at the "Nash" level \( (n) \), given by:

\[
n = \arg \max_x \beta x^{\frac{\sigma - 1}{\sigma}} - x = \beta^\sigma m,
\]

and the amount of good quality inputs in \( n \) in good matches and \( \theta n \) in bad matches. It follows that \( n < m \): as in any standard model, there is under investment. The ex-ante transfer reflects the amount of good quality inputs eventually provided, so that \( t = (1 - b + b\theta) (\beta R(n) - n) \), and the final good producer still captures the entire benefit of the relationship.\(^{13}\)

### 3.4 Sustaining cooperation

We now let the simple game of the previous subsection be repeated an infinite number of times, and show that the repeated interaction can incentivize a supplier in a good match to invest more than what his short term self-interest would require. We take as given the structure of the one shot interaction: ex-ante monetary transfers are paid just after the producers have chosen a supplier, suppliers undertake the investment, and revenues are shared according to ex-post Nash bargaining. The contractible case remains very simple: it is a repetition of the one-shot interaction. Normalized investment is always at the first best level \( m \). A newborn producer initially chooses a supplier at random, and keeps switching supplier every period until he finds a good match. Once he has found a good match, he sticks with him, and, because of Bertrand competition, the good match supplier offers an ex-ante transfer that allows him to capture the entire surplus of the ongoing relationship over any new relationship. In the noncontractible case, the same SPNE exists but the supplier’s normalized investment is set at \( n \), we will refer to this noncooperative SPNE as the “Nash case”. We now derive a “cooperative” SPNE.

We say that a supplier cooperates with his producer if he provides more (normalized) good quality inputs than he would in a one-shot interaction, that is if his normalized investment belongs to \( (n, m] \). Of course, there is a profusion of SPNE featuring some level of cooperation, and we are going to restrict attention to a particular class of equilibria. In every period, the producer has to choose between continuing a relationship or switching to a new partner. The difficulties in generating cooperation when producers can switch suppliers at will are well known. Classic solutions often involve some form of "collusive" behavior from the suppliers: for instance, in the first model of [Kranton, 1996], parties agree to a limited level of cooperation at the begin of a relationship, this behavior is "collusive" because in a new relationship, the supplier has an incentive to agree with the producer to both deviate to strategies allowing for full cooperation from the start of the relationship. In the context of innovation, which is an activity undertaken by a firm - sometimes an entrant - in order to increase its market share, we

\(^{13}\)Note that underinvestment not only decreases profits but further reduces welfare: in the full contractibility case, the monopoly distortion already leads to a production of differentiated goods lower than the welfare maximizing level, incomplete contractibility aggravates this initial monopoly distortion.
think that it is particularly important to avoid relying on such collusive equilibria, and to let pairs cooperate “as much as they can” from the beginning of the relationship. The presence of heterogenous matches in our model is what allows to sustain cooperation without resorting to “collusive” behavior by the suppliers. In fact, in her second model, [Kranton, 1996] considered an equilibrium robust to pairwise deviation where some suppliers had a higher discount rate, in our setting relationship-specific bad matches play the same role but avoid the introduction of adverse selection.

More specifically, every repetition of the game is composed of three moves: in phase 2 suppliers make their offer for the ex-ante transfer, in phase 3 producers choose a supplier, and in phase 6 suppliers undertake the investment. We say that a good match supplier suffered a "terminal deviation" with a producer if, either the supplier has at least once invested less than he was supposed to in a relationship with the same producer, or if the producer has already worked with another good match supplier since the first time he worked with the supplier. We denote by \( H_t (j, k) \), the set of histories of the game after \( t \) repetitions just before phase 6 where producer \( j \) has chosen to work for the period \( t \) with supplier \( k \). We then denote by \( H^d_t (j, k) \) the subset of histories \( H_t (j, k) \) where the supplier \( k \) is a good match for producer \( j \), but where a terminal deviation has occurred between producer \( j \) and supplier \( k \), by \( H^b_t (j, k) \) the subset of histories \( H_t (j, k) \) where the supplier \( k \) is a bad match for the producer \( j \), and by \( H^g_t (j, k) \) the subset of histories where the supplier \( k \) is a good match for producer \( j \) and no terminal deviation within the pair has occurred. We then restrict attention to symmetric SPNE with the following characteristics:

1. strategies of suppliers towards a producer depend only on their personal history with the producer and on whether the producer knows another supplier who is a good match and on whether that other supplier is one with whom a terminal deviation has occurred or not. Strategies of producers are independent on the history of the game with other producers (condition C1);

2. normalized investment with producer \( j \) by supplier \( k \) in a history belonging to \( H^d_t (j, k) \) is given by \( n \) (condition C2);

3. normalized investment with producer \( j \) by supplier \( k \) in a history belonging to \( H^b_t (j, k) \) is given by \( n \) (condition C3);

4. at any history \( h_t \in H^g_t (j, k) \), the strategies played by the producer \( j \), \( \sigma_j | h_t \), and the supplier \( k \), \( \sigma_k | h_t \), must be such that there is no pair of strategies for the producer and the supplier where \( \sigma' | h_t = (\sigma'_j | h_t, \sigma'_k | h_t, \sigma_{-k} | h_t) \) is also a SPNE (\( \sigma_{-k} | h_t \) denotes the strategies of the other suppliers), and where \( \sigma' | h_{t+\tau} \) also satisfies C4 for any \( h_{t+\tau} \in H^g_{t+\tau} (j, k) \) and \( \tau > 0 \) following history \( h_t \), such that \( u_j (\sigma' | h_t) + u_k (\sigma' | h_t) > u_j (\sigma | h_t) + u_k (\sigma | h_t) \), where \( u_j \) is the value of the producer and \( u_k \) of the supplier. (condition C4).
As is common in cooperating games where producers can start a new relationship, condition C1 restricts attention to strategies that depend on the "minimal" amount of information on the history of the game. Strategies played with different producers are entirely dependent of each other, and the only information that a supplier uses in a relationship with a producer is his own personal history with the producer and whether the producer knows a good match with whom no terminal deviation has occurred, a good match with whom a terminal deviation has occurred or no good matches. This is, in general, an essential information as it affects the outside option of the producer, and cannot be ignored (however strategies can be independent on whether the producer knows more than one other good match supplier). Condition C2 means that within a pair, players are using trigger strategies: a deviation is punished by reverting to the strategies of the one-shot game. It is a natural assumption in this context, and features the sharpest punishment within the pair ([Baker et al., 1994] or [Francois et al., 2003] use trigger strategies in similar contexts). An interpretation of C2 is that trust in the pair producer/supplier is forever broken if a deviation occurs: if a supplier deviates once, the producer is afraid the supplier will cheat again, therefore he no longer accepts an ex-ante transfer leaving any rent to the supplier, who, in return, keeps investing n; similarly, if a producer does not keep his good match and finds a new good match, the supplier is afraid the producer will keep reverting to his new good match in the next period and is not willing to invest more than n.\textsuperscript{14} 15

Condition C3 rules out cooperation in bad matches: producers in a bad match will look for a new supplier in the following period, which, in turn, deters cooperation. In Appendix A, we justify condition C3 by showing, first, that there is no SPNE satisfying C1, C2 and C4 at any node in $H_t(j,k) \setminus H_t^d(j,k)$, that is it is impossible to extend the equilibrium to bad matches as well; and, second, that when bad matches are sufficiently bad, there is no equilibrium where a single supplier would be willing to cooperate in a bad match. Note that even if bad matches were not to involve an actual loss in productivity the equilibrium we describe would still hold, what is essential, is that for some reason, cooperation is impossible in some matches. The productivity loss provides a reason for why this might be the case.\textsuperscript{16}

Finally, condition C4 means that strategies are designed such that parties in a good match and who have not experienced a terminal deviation, achieve the highest level of cooperation possible, knowing that in subsequent periods their strategies will also lead to the highest level of cooperation unless a terminal deviation has occurred. Condition C4 is the one that rules out situations where cooperation builds up progressively, or where all suppliers coordinate on

\textsuperscript{14}It is obviously impossible to generate cooperation if condition C4 were to also apply at histories $H_t^s(j,k)$.

\textsuperscript{15}Here a good match supplier forgives a producer who switches, but only finds out a bad match. That is, a good match supplier expects that the producer is willing to keep working with him, when the producer has not found a new good match after they started working together for the first time. This assumption is innocuous in the no innovation case, and we could as well assume that cooperation in the pair ceases forever if the producer switches, whether he finds a new good match or not.

\textsuperscript{16}An alternative story could be one where a fraction of the suppliers are irrationally bad, which is the kind of assumption often used in models of reputation [Mailath and Samuelson, 2006].
punishing a producer who has deviated.\textsuperscript{17, 18} Overall, the equilibrium we describe features the highest possible level of cooperation in good matches, with the fiercest competition amongst suppliers. In Appendix A, we prove:

**Proposition 1** Under conditions C1, C2, C3 and C4 payoffs in a symmetric SPNE are uniquely defined on equilibrium path. In particular, normalized investment is equal to $n$ in bad matches, and normalized investment is constant, equal to a unique $x^* \in (n, m]$, in good matches.

We specify below the level of investment $x^*$. Appendix A provides a full formal description of a SPNE that satisfies C1, C2, C3 and C4. Inversely, as shown in appendix A, any equilibrium satisfying C1, C2, C3 and C4 has the following structure:

1. there is Bertrand competition amongst suppliers: when a new relationship is formed, producers capture the entire expected value of the relationship, and when a supplier is a good match he captures the entire surplus of the relationship over any other relationship

2. producers keep switching suppliers until they find a good match supplier, once they have found one, the supplier cooperates and the producer keeps working with him;

3. if a good match supplier invests less than $x^*$, then, either the producer looks for a new supplier in the following period, or the pair keeps working together but normalized investment remains at $n$.\textsuperscript{19}

Whether, once a deviation has occurred, it is better for the producer to stick with a non cooperating good match, or to look for a new supplier, depends on parameter values. We now describe in more details the equilibrium in the later case. In particular, we show how the equilibrium normalized level of investment in good matches, $x^*$, is determined (Appendix A deals with the former case, which is very similar).\textsuperscript{20} We use $V^z_0$ (respectively, $V^z_1$), to denote the value of a producer ($z = p$), of a supplier ($z = s$), or their joint value ($z = T$), when they are in a new relationship, before the type of the match is revealed, (respectively, when they are in a good

\textsuperscript{17}The recursive definition of condition C4 is actually not necessary in the no innovation case (so that one can ignore the restriction that $\sigma'[h_{t+1}]$ also satisfies C4 in the definition of condition C4), but simplifies the characterization of the equilibrium in the innovation case.

\textsuperscript{18}A condition equivalent to condition C4, is that the equilibrium is renegotiation proof except after a terminal deviation, and the amount of investment is independent on the ex-ante transfer that has been paid (it is possible to build other equilibria renegotiation proof except after a terminal deviation, but they feature lower cooperation, and requires that if the supplier’s ex-ante transfer is higher than it is supposed to, the supplier would reduces the level of cooperation, basically punishing himself...).

\textsuperscript{19}If a producer does not keep a good match supplier, but finds a new good match supplier, he keeps working with the new good match supplier, whose normalized investment is given by $x^*$.

\textsuperscript{20}The following is not a proof of proposition 1, the proof is in Appendix A. Here we take as given the description of the SPNE and rederive the IC constraint.
match relationship). Since investment is constant in good matches, the value of a relationship remains constant after the first period. As already mentioned, Bertrand competition drives away any rent for the supplier at the beginning of a new relationship, but, ensures that a supplier in a good relationship captures the whole surplus of the good relationship over the second best option for the producer (starting a new relationship). Therefore, we get:

\[ V_s^0 = 0 \]  
\[ V_s^1 = V_T^1 - V_T^0 \text{ and } V_p^1 = V_p^0. \]  

The total value of a new relationship can be expressed as:

\[ V_T^0 = (1-b) \left( \Pi(x^*) + \frac{1-\delta^D}{1+\rho} V_T^1 \right) + b \left( \theta \Pi(n) + \frac{1-\delta^D}{1+\rho} V_0^T \right), \]  

with probability \((1-b)\), the match turns out to be good, profits are given by \(\Pi(x)\) and the value of the relationship next period is \(V_T^1\); with probability \(b\), the relationship turns out to be bad, per period profits are only given by \(\theta \Pi(n)\), the value of the supplier in the next period is 0, but the value of the producer is once again equal to the total value of a new relationship. The total value of a good match is given by:

\[ V_T^1 = \Pi(x^*) + \frac{1-\delta^D}{1+\rho} V_T^1, \]  

profits are always \(\Pi(x^*)\), and the relationship will continue in the following period. Combining (6), (7) and (8), we get:

\[ V_s^1 = \frac{(1+\rho) b (\Pi(x^*) - \theta \Pi(n))}{1+\rho - b (1-\delta^D)}, \]  

hence, the value of a good match supplier is the per period difference between profits in a good match and expected profits in a new relationship \(b (\Pi(x^*) - \theta \Pi(n))\), properly discounted.

When the supplier makes his investment decision, the ex-ante transfer is already paid, however, he is yet to receive the ex-post profits from the investment, that is a share \(\beta\) of the revenues minus the cost. Therefore, the supplier has an incentive to deviate from investing an amount \(x\), by investing the Nash level \(n\), which maximizes his ex post profits, his gain would then be given by \(\varphi(x)\), where:

\[ \varphi(x) \equiv (\beta R(n) - n) - (\beta R(x) - x). \]  

However, if the supplier carries out the investment \(x\), the producer will keep him in the following period, guaranteeing the supplier a value \(V_s^1\), whereas, if he does not, he will lose the supplier (this is the case we are considering here) and the value he gets out of the relationship will be 0. The supplier would then be willing to carry on the investment \(x\) only as long as:

\[ \varphi(x) \leq \frac{1-\delta^D}{1+\rho} V_s^1, \]  

18
In an equilibrium where cooperation is as high as possible, investment should maximize joint profits under this incentive constraint. Note, however, that $V_1$ itself depends on the equilibrium level of investment in good matches $x^*$, hence $x^*$ is a solution to a fixed point problem. In Appendix A, we prove that this fixed point problem defines a unique level of cooperation $x^*$:

**Proposition 2** When the value of a new relationship exceeds the value of a good match where the supplier does not cooperate, the equilibrium amount of investment $x^*$ in good matches is given by the first best level $m$ if

$$
\varphi (m) \leq \frac{1 - \delta^D}{1 + \rho - b(1 - \delta^D)} b (\Pi (m) - \theta \Pi (n)),
$$

and otherwise, by the unique solution in $(n,m)$ to:

$$
\varphi (x^*) = \frac{1 - \delta^D}{1 + \rho - b(1 - \delta^D)} b (\Pi (x^*) - \theta \Pi (n)).
$$

Profits in a good match weakly increase with the number of bad matches, $b$, and decrease with the relative productivity of bad matches, $\theta$, the discount rate, $\rho$, and the probability of death $\delta^D$.

How much suppliers cooperate depends on how bad the alternative option is. Therefore if the probability of a bad match, $b$ is higher, or if they are more severe (low $\theta$), a good relationship will have more value, and the potential for cooperation is higher. A higher value of the future (lower $\rho$ and $\delta^D$) have the same effect. As shown in Appendix A, our analysis does not depend on the assumption that the value of a new relationship exceeds the value of a noncooperative good match.\footnote{More specifically the level of equilibrium derived so far holds as long as $\left(1 + \frac{b(\rho + \delta^D)(1 - \theta)}{(1 - b)(1 + \rho)} \right) \Pi (n) \leq \Pi (x^*)$. Otherwise $x$ is at the first best if $\varphi (m) \leq \frac{1 - \delta^D}{1 + \rho + \delta^D} (\Pi (m) - \Pi (n))$, and otherwise the unique solution in $(n,m)$ to $\varphi (x^*) = \frac{1 - \delta^D}{1 + \rho + \delta^D} (\Pi (x) - \Pi (n))$. In this case, $x^*$ is determined by the difference in profits between a good match on path and when a deviation has occurred, and does not depend on the number or severity of bad matches.}

In this equilibrium cooperation arises in good matches, because, in the following period, the supplier captures the additional value of the ongoing relationship over a random new relationship. This specific features comes from the assumption of Bertrand competition, however, this equilibrium can be generalized to the case of ex-ante Nash Bargaining between suppliers and producers, or even to the case where producers make take it or leave it offers to suppliers, by letting producers offer a bonus to the supplier when they deliver the appropriate amount of good quality inputs. Producers would themselves face an incentive compatibility constraint, and would pay the bonus only if it remains smaller than the cost of finding a new supplier. Equilibrium level of investment are then exactly identical, but the description of the equilibrium is slightly more complicated.
The incumbent supplier has the informational advantage that the nature of the match has been revealed. This informational advantage acts as a fixed cost that pushes the producer to stick to the same supplier, which, in return, incentivizes the supplier to cooperate. Crucially, this fixed cost interacts naturally with the incomplete contractibility: in a situation with incomplete contractibility, there is no cooperation in bad matches, as bad matches have no prospect, which makes bad matches “even worse” relatively to good matches than in the complete contractibility case or the no cooperation case.\textsuperscript{22} This feature will be crucial when we introduce innovation.

4 Stickiness of relationships and innovation

The previous section established that cooperation in long term relationships mitigates the under-investment problem associated with contractual incompleteness. We now turn to the downside of this, namely that cooperation makes relationships “too rigid”, which creates dynamic inefficiencies. Dynamic inefficiencies could arise for a large variety of macroeconomic issues (for example when the economy has to adapt to business cycles shocks), but, in this paper we specifically focus on innovation, and consider an endogenous growth model in which suppliers engage in research to improve technology. In the first subsection, we analyze whether a producer in a good relationship would be willing or not to switch to a new innovator, and establish that cooperation makes it harder for an innovator to break into the market; in the second subsection, we analyze how innovation itself affects the degree of cooperation; in the third subsection, we show that the incentive to innovate and therefore growth, are reduced with noncontractibility, and may be further reduced by cooperation, because of the rigidity of relationships; finally, we show through simulations, that welfare might be reduced by cooperation.

4.1 To switch or not to switch

In this subsection we study the trade-off faced by a producer in a good match when an innovator comes into the market with a superior technology: should he stay with his current good match, or should he switch to the innovator, bearing the risk that the innovator will be a bad match. We assume that every period, an outside supplier has the opportunity to innovate. For the moment, we take the innovation decision as given and assume that an innovation happens with probability $\delta^I$ (we show how $\delta^I$ is determined in subsection (4.3)). When an innovation occurs, the innovator has access to a technology $\gamma$ times more productive than the previous frontier technology, however, we assume that after a single period, the innovator is imitated, and all

\textsuperscript{22}Even if cooperation were to arise in a specific bad match, the level of cooperation - defined by the normalized investment level - would be lower than in good matches, and so even this “cooperative” bad match would be relatively worse, than in the complete contractibility case or the no-cooperation case.
suppliers have access to his technology (in subsection (5.4), we analyze what happens when patents last for more than one period). We denote by \( A \) the frontier technology level, so that, in periods without innovation all suppliers use technology \( A \), and, in periods with innovation only the innovator uses the technology \( A \), while the other suppliers use \( \gamma^{-1}A \).\(^{23}\)

The overall structure of the game remains the same as in section 3. In the contractible case, the description of the equilibrium remains simple. Normalized investment is always at the first best level \( m \). Producers switch suppliers until they find a good match, if they have not found a good match and if an innovation occurs, they try out the innovator first (he has the same probability of being a good match as any other suppliers but he offers a more productive technology). If they have found a good match, they face a trade-off, switching to the innovator entails a better technology but the risk of facing a bad match. However, the technological advantage of the innovator lasts for only one period and, if the innovator turns out to be a bad match, the producer can always revert to his old supplier (who remains a good match). Therefore, the producer switches to the innovator if and only if the expected productivity of the innovator is higher than the productivity of an outdated good match, that is, if and only if:

\[
1 - b + b\theta - \gamma^{-1} > 0, \text{ or equivalently, } \gamma > \gamma^C \equiv (1 - b + b\theta)^{-1},
\]

with probability \((1 - b)\) the innovator is a good match, with probability \( b \) he is a bad match, but the technology of the old good match supplier is \( \gamma \) times less productive. The Nash case is exactly identical except that normalized investment are always at the Nash level \( n \), so that producer previously in a good match switch to the innovator if and only if \( \gamma > \gamma^{Nash} = \gamma^C \).

For the cooperative case, we extend naturally the requirement on the SPNE: conditions C2, C3 and C4 hold unchanged, condition C1 should allow for strategies to depend on whether the period is one with innovation or not. Note, however, that the fact that a good match supplier “forgives” a producer who switches suppliers but does not find a new good match, is no longer innocuous in periods where innovation occurs. Indeed, in this set-up, cooperation between a producer and an old good match supplier, resumes after one period if the producer switches to the innovator for one period, and the innovator turns out to be a bad match. As shown in subsection (5.2), the main results of the section are actually reinforced in the alternative case, where suppliers do not forgive producers who switch, no matter whether they have found good or bad matches, but the current version allows for simpler expressions.\(^{24}\) Therefore, producers

\(^{23}\)Of course, once growth is introduced the differentiated sector will eventually become so productive, that the consumption of the homogenous good will be driven to 0. Technically, what we present here is an approximation, which is valid only as long as the productivity of the differentiated sector remains sufficiently low. Alternatively, we can assume that the productivity of the homogenous good grows at the rate of the technological frontier (through knowledge externality), in which case, what we present is not an approximation but the exact solution. We also assume that \( \delta^I \) and \( \gamma \) are sufficiently small, that the values of firms converge.

\(^{24}\)The recursive formulation of condition C4 now plays a role. Under some parameter values, at some node, a higher level of cooperation at could be reached if the supplier’s strategy was to punish a producer who switches to the innovator even if the innovator were a bad match: so that there would not be any SPNE satisfying C1,
switch suppliers, choosing the innovator whenever innovation occurs, until they find a good match. Once they have found a good match, they keep working with the same supplier as long as no innovation occurs. If an innovation occurs, they face a trade-off between switching or not, knowing that, if they switch, and the innovator turns out to be a bad match, they can go back to their old good match supplier in the following period, and the old good match supplier will resume cooperation.  

Because of Bertrand competition, a producer previously in a good match relationship switches to the innovator, if and only if the expected value of joint profits with the innovator is higher than with the old supplier. As the old supplier imitates the innovator’s technology after one period, and as the producer can resume cooperation with the old supplier if the innovator turns out to be a bad match, the decision to switch depends only on the difference in expected profits in the first period. The innovator is a good match with probability $(1 - b)$, in which case he invests $x^*$, and a bad match with probability $b$, in which case he invests $n$, while the old good match invests $y^*$ and his technology is $\gamma$ times less productive, therefore:

**Lemma 1** Producers previously in a good match switch to the innovator if and only if

$$(1 - b) \Pi (x^*) + b \theta \Pi (n) - \gamma^{-1} \Pi (y^*) > 0. \quad (13)$$

**Proof.** See Appendix C  

We can rewrite (13) as:

$$1 - b + b \theta (\Pi (n) / \Pi (x^*)) - \gamma^{-1} (\Pi (y^*) / \Pi (x^*)) > 0. \quad (14)$$

Now as shown in Appendix C, cooperation occurs in good matches so that $x^* > n$, moreover, as explained below, $y^* \geq x^*$, therefore $\Pi (n) / \Pi (x^*) < 1$ and, $\Pi (y^*) / \Pi (x^*) \geq 1$, so that (12) is more easily satisfied than (14), which gives us:

**Proposition 3** (i) The parameter set for which innovators capture the whole market in the cooperative case is strictly smaller than the parameter set for which innovators capture the whole market in the contractible or the Nash cases. (ii) In particular, the minimum technological leap required for an innovator to capture the whole market in the cooperative case ($\gamma^{NC}$) is higher than that in the contractible or Nash cases ($\gamma^{C}, \gamma^{Nash}$): $\gamma^{NC} > \gamma^{C} = \gamma^{Nash}$.  

Moreover, as shown in Appendix D:

\[\text{C2, C3 and a non recursive C4.}\]

\[\text{25As in the previous section, when a supplier deviates and invests less than he is supposed to, there are two possible cases depending on parameter values, either the producer starts looking for a new supplier, or the producer stays with the same supplier who does not cooperate anymore.}\]

\[\text{26Technically, $\gamma^{NC}$ is defined as the infimum on the range of $\gamma$ for which an innovator can capture the entire market and make strictly positive profits.}\]
Remark 1 Consider the case where the value of a new relationship is larger than the value of a good match relationship with no cooperation. Then, the levels of investments \((x^*, y^*)\) are uniquely defined. For \(\gamma < \gamma^{NC}\), innovation is not adopted by producers previously in a good match, and there is an \(\eta > 0\), such that for \(\gamma \in (\gamma^{NC}, \gamma^{NC} + \eta)\), innovation must be adopted by all producers, for \(\gamma = \gamma^{NC}\), the share of producers previously in a good match who adopt the innovation is undetermined.

Proposition (3) delivers the first important message of the paper: in a context of weak contractibility, cooperation makes it more difficult to break down existing relationships. Because of the existence of bad matches, for \(\gamma\) sufficiently close to 1, innovations are not adopted by suppliers in good matches, but the threshold for adoption is higher in the cooperative case than in the contractible or Nash cases.

The intuition behind this result arises from two effects. The first - and most robust - effect is a worse bad matches effect: a bad match is more costly relative to a good match in the cooperative case, indeed, in this case, bad matches do not only involve an inherently lower productivity level, they also involve less investment, this effect is reflected by the term \(\Pi(n)/\Pi(x^*)\) in (14). The second effect is an encouragement effect, namely the fact that cooperation is (weakly) higher when using the outdated technology, than it is when using the frontier technology (that is, \(y^* \geq x^*\), which affects the decision to switch or not through the term \(\Pi(y^*)/\Pi(x^*)\) in (14)). The reason is that the incentive to deviate for a given investment level is higher for a supplier using the frontier technology than for a supplier using the outdated technology, whereas the reward from cooperation is the same. Indeed, the incentive to deviate is scalable by the technology used by the supplier, so that it is given by \(\varphi(.)\gamma^{-1}A\) for suppliers using the outdated technology, whereas it is given by \(\varphi(.)A\) for suppliers. The reward from cooperation is the same because, in the following period, the outdated suppliers have access to the frontier technology as well, so that the values of all suppliers are the same, irrespective of whether they were outdated or not in the previous period, and the value of a supplier is precisely the reward from cooperation. In other words, the opportunity to imitate the frontier technology in the following period encourage outdated suppliers to provide a larger effort, partly compensating the fact that they are using an outdated technology.\(^{27}\)

As the levels of investment \(x^*\) and \(y^*\) are endogenous, our description of the equilibrium could leave room for potentially multiple equilibria (under the set of strategies considered so far), remark (1) shows, that this does not occur when producers would rather find a new supplier than stay with a noncooperative good match. Moreover, note that \(\gamma^{NC}\) depends on the rate of innovation \(\delta^I\) through \((x^*, y^*)\), but because \(\delta^I\) does not, proposition 3 (ii) would

\(^{27}\)This latter effect is very strong here as patents last for only one period, however, some form of this effect will always be present as long as the supplier has a positive probability to eventually get access to the frontier technology.
remain true with endogenous innovation and different rates of innovation for the cooperative, Nash and contractible cases.

That good match suppliers may be more reluctant to adopt the technology of the innovator when cooperation arises in long-term relationships, does not directly reduce welfare, it does so only through the impact on the decision to innovate (see subsection (4.3)).\footnote{Actually, from a welfare point of view, and at given rate of innovation, producers switch to the innovator “too much”. Indeed, bad matches are even more detrimental to welfare than to profits, as final good producers are monopolists (the level of normalized investment that maximizes welfare is higher than \( m \)), and switching to the innovator inevitably involve more bad matches.} Nevertheless, proposition (3) (ii) states that innovation would be immediately adopted by all firms when \( \gamma \in (\gamma^C, \gamma^{NC}) \) in the contractible and Nash cases but not the cooperative case, so that we get:

**Corollary 1** When innovation \( \gamma \in (\gamma^C, \gamma^{NC}) \), there is more technological differences across firms in the cooperative case than in the contractible or Nash cases.

The model predicts that there should be more technological differences and/or that innovations spread slower in countries with poor contractibility institutions (as long as cooperation arises in equilibrium), because \( \gamma^C \) is independent of \( \delta^I \).

### 4.2 Impact of innovation on the level of cooperation

As in the no innovation case, a lower discount rate, \( \rho \), a lower probability of death \( \delta^D \), a larger probability of a bad match \( b \), and a lower productivity in bad matches \( \theta \) favor cooperation (that is, increase the levels of normalized investments \( x^* \) and \( y^* \)). We now briefly look at the impact of innovation itself (the size \( \gamma \), and the rate \( \delta^I \)) on the level of cooperation, and we get:

**Remark 2** Consider the case where the value of a new relationships is higher then the value of a relationship with a noncooperative good match, and where the innovator captures the entire market, then the level of cooperation \( (x^*, y^*) \) is increasing in the size of innovations \( \gamma \), and, for sufficiently small \( \gamma (\gamma b (1 - \delta^D)(2 - \delta^I) < 1 + \rho) \), is decreasing in the rate of innovation \( \delta^I \).

**Proof.** See Appendix E  ■

Therefore, when the innovator captures the entire market, scarce but large innovations (large \( \gamma \), high \( \delta \)) favor cooperation in good matches. Larger innovations lead to a higher growth rate, which increases the expected value a supplier can capture by cooperating, favoring more investment in good matches. More frequent innovations have three effects on the investment levels: (i) a positive effect through a higher growth rate, (ii) a negative effect through a higher probability of ending the relationship, and (iii) a further negative effect that comes from the fact that the benefit of being in a good match over a random match is higher in periods with no innovation, than in periods with innovation, knowing that, this benefit is precisely what drives the incentive to cooperate. For sufficiently small innovations, the effect (ii) dominates.
the effect (i), and therefore more frequent innovations will lower the level of cooperation. We can compare this result to [Francois et al., 2003], who show that an increase in innovation can push firms towards providing short-term contract arrangements instead of implicit guarantees of lifetime employment to their workers. In our model, the same idea is captured by the decrease in cooperation following an increase in the rate of innovation.29

4.3 Endogenous rate of innovation

Subsection (4.1) has showed that cooperation creates rigidities in long-term relationships, we now show that these rigidities are the source of dynamic inefficiencies, by studying how the equilibrium rate of innovation \( \delta^I \) is determined. Every period one supplier gets a new idea. This idea turns into a useful innovation with probability \( \delta^I \) if the potential innovator invests \( \psi (\delta^I) A \) (where \( A \) is the frontier technological level before innovation occurs), where \( \psi \) is a convex function and where \( \lim_{\delta^I \to 1} \psi' (\delta^I) = \infty \) (the size of innovation \( \gamma \) is a constant). Because the probability that the potential innovator has already made a successful innovation is infinitesimal, the market share of the potential innovator is infinitesimal, so that, for all purposes the potential innovator is an entrant. In this subsection, we compare the rate of innovation in the three different cases: contractible, Nash and cooperative.

Because of Bertrand competition, when the relationship with the innovator is the highest possible, the innovator captures the entire benefit of this relationship over the second best option of the producer.30 Recall that patents last for only one period, and that, in the cooperative noncontractible case, good match suppliers resume cooperation if the producer switches to the innovator and the innovator turns out to be a bad match. The difference between the value of a relationship with the innovator and the value of a relationship with the best alternative, is then simply equal to the difference in profits in the first period. We denote by \( V_{s,t}^{I,K} \) the value captured by the innovator (normalized by the after innovation frontier productivity level) from a relationship with a producer, who knows a good match supplier \( t = g \), or who does not know any good match supplier \( t = b \), for the contractible case \( K = C \), the Nash case \( K = Nash \) and for the cooperative case \( K = NC \). Defining \( X^+ = \max (X, 0) \), we get that, in the contractible case, the value captured by the innovator from a relationship with a producer previously not in a good match is given by:

\[
V_{s,b}^{I,C} = (1 - b + b\theta) (1 - \gamma^{-1}) \Pi (m), \tag{15}
\]

while the value captured by the innovator from a relationship with a producer previously in a

\[29\text{In their model the size of innovation varies across sectors, and sectors with the smallest size of innovation are the ones supporting long-term implicit contracts.}
\]

\[30\text{If instead of Bertrand competition, we had assumed ex-ante Nash Bargaining, the innovator would capture only part of the difference, but as long as he captures a positive part, the results of this subsection carry through.}
\]
good match is given by:
\[ V_{s,g}^{s,b} = (1 - b + b\theta - \gamma^{-1})^+ \Pi (m). \] (16)

The situation of producers previously in a good match has been analyzed in (12), for producers previously not in a good match, the reasoning is similar: joint expected profits are the same with the innovator and any other supplier, except in the first period where they are \( \gamma \) times higher with the innovator, Bertrand competition allows the innovator to capture all the surplus of a relationship with him over any other relationship. Similarly, for the Nash case, we get:
\[ V_{s,b}^{s,b} = (1 - b + b\theta) (1 - \gamma^{-1}) \Pi (n) \quad \text{and} \quad V_{s,g}^{s,g} = (1 - b + b\theta - \gamma^{-1})^+ \Pi (n). \] (17)

Finally, in the cooperative case, we get:
\[ V_{s,b}^{s,b} = (1 - b) (\Pi (x^*) - \gamma^{-1}\Pi (y^*)) + b\theta (1 - \gamma^{-1}) \Pi (n) \] (18)
\[ \text{and} \quad V_{s,g}^{s,g} = ((1 - b) \Pi (x^*) + b\theta \Pi (n) - \gamma^{-1}\Pi (y^*))^+. \] (19)

The case of producers previously in good matches was analyzed in (13). The case of producers previously not in good matches follows the same logic, if they choose a random outdated supplier instead of the innovator, expected joint profits will be the same except in the first period where the outdated supplier will be \( \gamma \) times less productive, and will invest \( y^* \) instead of \( x^* \) if he is a good match, because of Bertrand competition, the innovator captures the difference in expected joint profits. It can then be shown:

**Lemma 2** The value captured by the innovator from a relationship is lower in the cooperative case than in the contractible case: \( V_{s,b}^{s,b} > V_{s,b}^{s,b} \) and \( V_{s,g}^{s,g} \geq V_{s,g}^{s,g} \), however, it may be higher or lower than in the Nash case.

Three effects explain this result: the worse bad match effect (bad matches in the cooperative noncontractible case feature lower investment in addition to lower productivity compared to good matches), the encouragement effect (in the cooperative noncontractible case, outdated suppliers are willing to cooperate more than suppliers at the technological frontier), and a scale effect. By scale effect, we refer to the fact that thanks to cooperation, underinvestment in good matches is less severe than in the noncooperative case, therefore, profits are higher than without cooperation, but they remain (weakly) lower than in the contractible case, that is: \( \Pi (n) < \Pi (x^*) \leq \Pi (m) \). The first two effects lower the value that the innovator captures in the cooperative case compared to the contractible and Nash cases. The third effect increases the value captured by the innovator compared to the Nash case, but further decreases it compared to the contractible case. Therefore in the comparison between the contractible and cooperative cases, the three effects work in the same direction, whereas in the comparison between the contractible and the Nash cases, the scale effect works in the opposite direction to the worse bad matches and encouragement effects.
In equilibrium, the steady-state fraction of firms previously not in a good match is constant, independent of the rate of innovation and given by $\omega = \frac{\delta^D}{1 - (1 - \delta^D) b}$.

Hence, assuming that the steady state has been reached, the innovator solves the problem:

$$\max_{\delta} \gamma \delta \left[ \omega V_{I,K}^{s,b} (\delta^I) + (1 - \omega) V_{I,K}^{s,g} (\delta^I) \right] - \psi (\delta),$$

for $K = C, \text{Nash, NC}$. The first order condition uniquely defines the equilibrium rate of innovation in the contractible case ($\delta^C$), and in the Nash case ($\delta^{\text{Nash}}$). In the cooperative case, the value of the innovator depends on the equilibrium rate of innovation, so any fixed point of the first order condition would be a solution to the problem, we pick one solution and denote it $\delta^{NC}$.

From lemma 2, the value of the innovator at a given rate of innovation is always higher in the contractible case than in the noncontractible cases, but may be higher in the Nash case than in the cooperative case, therefore:

**Proposition 4** The highest equilibrium rate of innovation in the cooperative case is lower than the rate of innovation in the contractible case, and may even be lower than the rate of innovation in the Nash case: $\delta^{NC}, \delta^{\text{Nash}} < \delta^C$ but $\delta^{NC} \leq \delta^{\text{Nash}}$. Similarly, welfare is always lower with incomplete contractibility than with complete contractibility, and cooperation may increase or decrease welfare.

This proposition is the second important result of the paper: cooperation in long-term relationships can actually decrease the rate of innovation, and therefore, even welfare (despite the positive effect of cooperation on investments by suppliers). On one hand, cooperation pushes towards more innovation through the scale effect, but, on the other hand, the rigidities in relationships created by cooperation reduces innovation. Note that it is because of the knowledge externalities that cooperation can reduce welfare (if the rate of innovation was exogenously fixed - for instance at the social optimum - cooperation would necessarily be welfare enhancing).

Interestingly, when cooperation is very common, the risk that it reduces welfare is actually quite large. When $\delta^D$ is small, most producers have found a good match supplier who cooperates with them. However, in this case, $\omega$ is small, so the value of innovation is mostly given by the value captured from relationships with producers previously in good matches, but, cooperation is more likely to have a negative effect on the value captured from relationships with producers previously in good matches than from relationships with producers previously not in good matches, that is, it is more likely that $V_{I,NC}^{s,g} < V_{I,Nash}^{s,g}$ than that $V_{I,NC}^{s,b} < V_{I,Nash}^{s,b}$, because the worse bad match effect plays no role for producers previously not in a good match.

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31 $\omega$ is the share of firms that know a good match supplier willing to cooperate with them. It does not depend on the rate of innovation, because when an innovation occurs, producers do not lose the possibility to cooperate with their old supplier. On the contrary the expected number of firms in good matches in a given period will depend on the rate of innovation, when the innovator captures the entire market.

32 We could also assume that $\psi$ is sufficiently convex to rule out this possibility.
Now, note that if \( \gamma \in (\gamma^C, \gamma^{NC}) \), relationships never break down in the cooperative case but do so in the contractible case, if \( \gamma > \gamma^{NC} \), innovations break up relationships in both cases, but as innovations are more frequent in the contractible case, relationships still last longer in the cooperative case, therefore:

**Corollary 2** Relationships last longer in the cooperative case than in the contractible case

Therefore the model predicts that as long as cooperation occurs, relationships should last longer in countries with poor contractibility institutions.

### 4.4 Numerical exercise

To illustrate proposition 4, we simulate our model and show one case where cooperation raises welfare, and one where it does not. Throughout we keep the following parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount rate ( \rho )</td>
<td>0.03</td>
</tr>
<tr>
<td>elasticity of demand ( \sigma )</td>
<td>2</td>
</tr>
<tr>
<td>productivity of bad matches ( \theta )</td>
<td>0.9</td>
</tr>
<tr>
<td>death rate of producer ( \delta^T )</td>
<td>0.01</td>
</tr>
<tr>
<td>initial technological level ( A_0 )</td>
<td>0.01</td>
</tr>
<tr>
<td>bargaining share ( \beta )</td>
<td>0.5</td>
</tr>
</tbody>
</table>

We consider two values for the size of innovation \( \gamma \): \( \gamma_L = 1.1 \) and \( \gamma_H = 1.25 \), and two values for the share of bad matches \( b \): \( b_L = 0.1 \) and \( b_H = 0.33 \). The cost of innovation function takes the form \( \psi(\delta^I) = \psi \cdot (\delta^I)^2 / (1 - \delta^I) \), where \( \psi \) is adjusted so that the rate of innovation in the contractible case is given by \( \delta^C = 0.11 \), when \( \gamma = \gamma_H \) for both values of \( b \). For \( b = b_H \), we get \( \gamma^C \simeq 1.03 \) and as long as \( \delta^I \in (0, \delta^C) \), \( \gamma^{NC} \) is nearly constant equal to 1.12, therefore \( \gamma_L \in (\gamma^C, \gamma^{NC}) \) and \( \gamma_H > \gamma^{NC} \). Similarly, for \( b = b_L \), \( \gamma^C \simeq 1.01 \) and for the relevant range of \( \delta^I \), \( \gamma^{NC} \) is nearly constant equal to 1.02, so that \( \gamma_H > \gamma_L > \gamma^{NC} \). The following table compares the three cases: contractible, Nash and cooperative. The measure we use to express the welfare cost is the equivalent reduction in units of homogenous good consumed every period from the full contractible case (recall that every period a mass one of homogenous good is produced). For the Nash and cooperative cases, we first compute the welfare cost assuming that the rate of innovation is exogenously fixed at \( \delta^C \), and then endogenize the growth rate and compute the corresponding welfare cost.\(^{33}\)

\(^{33}\)For the parameter values chosen, the rate of innovation in the cooperative case is always unique, and no single pair in a bad match would be willing to start cooperating.
The most interesting part in table 1 is the last row. In this case, cooperation dramatically reduces the welfare cost of incomplete contractibility when innovation is exogenous (it decreases from 3.83% to 1.01%). However, when innovation is endogenized, the rate of innovation decreases a lot, and does so even more in the cooperative case than in the Nash case. As a consequence, the welfare costs of incomplete contractibility are much larger, and become even larger in the cooperative case than in the Nash case: cooperation reduces innovation so much that the static gains are dominated by the dynamic losses. When $\gamma = \gamma_H$ and $b = b_L$, the probability of a bad match is sufficiently low that the worse bad match effect (and the encouragement effect) are dominated by the scale effect, so that the rate of innovation is higher in the cooperative than in the Nash cases (and welfare of course is higher in the cooperative than in the Nash case). When $\gamma = \gamma_L$ and $b = b_H$, $\gamma < \gamma^NC$, and innovation does not break down existing good match relationships in the cooperative case, and innovation nearly disappears.

## 5 Extensions

In the previous section, we showed that cooperation in long-term relationships turns the static problem of under-investment into a dynamic problem of insufficient innovation. We now turn to different extensions of our basic model to assess the robustness of our results and get additional predictions.

### 5.1 Cooperation and distance to the technological frontier

In the version of the model presented so far, technological progress in a country occurred only through innovation. However, when a country is far from the world technological frontier,
the superior technology can progressively diffuse in the economy through imitation. One can argue that imitating an existing technology is a less risky activity than innovation and requires less special skills (see for instance [Acemoglu et al., 2006]), in this subsection we take the extreme view that at the beginning of every period all suppliers can freely imitate world the technological frontier. We then show that far from the world technological frontier cooperation in long-term relationships is a much better alternative to contractibility than it is close to it. The reason is twofold: first imitation is very important far from the world technological frontier and innovation becomes relatively less important in the overall growth process (if one country innovates but the other one does not, the second country will imitate more in the next period, partly compensating for the absence of innovation); second far from the technological frontier cooperation in long-term relationships is enhanced, that is the level of normalized investments in equilibrium are not constant any more and tend to be higher when a country is far from the world technological frontier. Indeed, the incentive to deviate in a given period is scaled by the current technology, while the reward from cooperation in the following period is scaled up by the future technologies, if growth is larger, the reward from cooperation becomes large relative to the incentive to deviate, allowing to sustain a higher level of cooperation in the first place. In effect, the effect of a higher growth rate is similar to the effect of a lower discount rate.

To illustrate this result we simulate numerically the economy when imitation from a technological frontier. To simplify things we consider a fixed technological frontier, $\overline{A}$, and we assume that at the beginning of every period, before any potential innovation occurs, all suppliers have access to a technology $A^b_t$, where $A^b_t$ satisfies:

$$A^b_t = A_{t-1} + \varepsilon \left| \overline{A} - A_{t-1} \right|,$$

$\varepsilon$ is the rate of imitation and $A_{t-1}$ is the final level of technology in period $t-1$. The final level of technology in period $t$ is given by $A_t = \gamma A^b_t$ if innovation occurs, and by $A_t = A^b_t$ if no innovation occurs, where the cost of innovation is given by $\psi (\delta^I) A^b_t$. For the numerical simulation we choose the same parameters as in 4.4 with $\gamma = \gamma_H$ and $b = b_H$, except for the initial level of technology that we fix at $A_0 = 0.0001$. We choose the level of the technological frontier at $\overline{A} = 0.01$ and the rate of imitation is $\varepsilon = 0.03$. Figure 1 shows the normalized level of investment when using the most advanced technology in good matches ($x^*_t$) (figure 1A), the rate of innovation (figure 1B), the level of the technology (figure 1C), and the utility flow minus 1 (figure 1D), for the three cases: contractible, cooperative and Nash.\(^{34}\) Figures 1C and

---

\(^{34}\)To model innovation in a consistent way, we draw a random number each period uniformly on $(0,1)$, if the number is smaller than $\delta^C$, innovation occurs in the contractible case, if it is smaller than $\delta^{Nash}$, it also occurs in the Nash case and if it is smaller than $\delta^{NC}$ it occurs in the cooperative case.
1D are on a logarithmic scale.

Figure 1: Innovation and Imitation

Figure 1A shows that the level of cooperation in the cooperative case is higher at the beginning, when the technology is far from the world frontier, the first best is even reached in the first period. Figure 1B shows that the rate of innovation itself is slightly higher, however, this is not a general result, it depends on the relative importance of a stronger scale effect (as cooperation is enhanced) and a weaker encouragement effect (the encouragement effect disappears initially as the $x_t^*$ and $y_t^*$ are both close to the first best) pushing towards a larger rate of innovation, versus a larger worse bad match effect pushing towards a smaller rate of innovation. Figure 1C shows that in the imitation phase the technologies of the three regimes are very close, the world technological frontier is achieved after 26 periods in the cooperative case and slightly sooner in the two other cases, thanks to an innovation that takes place in these two cases but not in the cooperative case. Afterwards technology grows the fastest in the contractible case and faster in the Nash than the cooperative case, according to the pattern implied by the steady-state rates of innovation. Figure 1D shows that the utility flow in the cooperative case and in the contractible case are very similar early on (far from the technological frontier), while the utility flow in the Nash case is much lower. However, far
from the technological frontier, cooperation in long term relationships ceases to be a good substitute to contractibility, and in this case where the equilibrium rate of innovation is lower in the cooperative than in the Nash case, eventually turns out to be a burden (the utility flow in the Nash cases ends up higher than in the cooperative case after period 150).

Therefore, this subsection showed that far from the world technological frontier, cooperation in long-term relationships can be a good substitute to contractibility, as long as imitation takes place easily in existing relationships. This reconciles our motivational evidence that showed the strong impact of contractibility on innovation with [Acemoglu and Johnson, 2005] who showed that contractibility seemed to have a moderate impact on the level of development (their focus being on former European colonies, the sample is composed of mostly developing countries, far from the technological frontier).

5.2 Case where cooperation ends forever when a producer switches supplier

So far, we made the assumption that a supplier forgives the producer if the producer switches supplier but only find a bad match. In this subsection we assume that a supplier stops cooperating if a producer switches to another supplier, no matter whether this other supplier is a good or a bad match. In particular, this means that if a producer switches to the innovator, and the innovator turns out to be a bad match, the old good match supplier would not be willing to resume cooperation if the producer comes back to work with him again.

**Loss of cooperation effect.** Therefore, the producer will suffer additional losses in the periods following innovation as he would have to either stick with a noncooperative good match or keep looking for a good match. This *loss of cooperation effect* is an additional force that pushes towards more rigid relationships and eventually a lower rate of innovation in the cooperative case. More specifically, as shown in Appendix G, in the case where producers would rather look for a new supplier than stay with a good match supplier who invests the Nash level, producers previously in a good match switch to the innovator if and only if:

\[
1 - b + b\theta \frac{\Pi(n)}{\Pi(x^*)} - \gamma^{-1} \frac{\Pi(y^*)}{\Pi(x^*)} - \frac{(1 - \delta^D) b^2 \left( (1 - \delta^I) \left( 1 - \theta \frac{\Pi(n)}{\Pi(x^*)} \right) + \delta^I \left( \frac{\Pi(y^*)}{\Pi(x^*)} - \theta \frac{\Pi(n)}{\Pi(x^*)} \right) \right)}{1 + \rho - b (1 - \delta^D) (1 - \delta^I + \delta^I \gamma)} > 0.
\]  

The last term in (21) (which is absent in (14)) reflects the loss of cooperation effect when producers prefer looking for a new supplier than staying with a noncooperative good match supplier. It is equal to the loss in expected profits from the risk of having to look for a new supplier in the subsequent periods, scaled by profits in a good match when no innovation arises (\(\Pi(x^*)\)).

Our main results completely carry through under this assumption: one

\[\text{35}\text{The term is easy to interpret: when a producer is looking for a new supplier (which happens only if the innovator turned to be a bad match, that is with probability } b\text{), there is a probability } b\text{ that the new supplier}\]
needs a larger $\gamma$ in the cooperative case to break up existing relationships than in the Nash or contractible cases, innovation is lower in the cooperative case than in the contractible case, and may be even lower than in the Nash case.\footnote{It is difficult to make a claim comparing the threshold $\gamma_{NC}$ in this case relative to the baseline. Indeed, the loss of cooperation effect pushes towards a much higher $\gamma_{NC}$ here, however, cooperation in general is lower than in the baseline reducing the impact of the worse bad match effect.} \footnote{Yet another possible assumption would be one where the nature of a match would be forgotten when a supplier and a producer stop working together. In this case, an effect equivalent to the loss of cooperation effect would arise both in the Nash and contractible cases. However, the effect would remain stronger in the cooperative case, and our results still carry through.}

**Multiple equilibria in the cooperative case.** Furthermore, in the cooperative case, when innovations are sufficiently large that all producers switch to the innovator, $\omega$, the share of producers who are not in an ongoing good match relationship (without deviation), depends positively on the rate of innovation: as more innovation means that more good match relationships break up definitely.\footnote{When producers would rather look for a new supplier than stay with a noncooperative good match, the share of producers previously not in a good match is given by $\frac{\delta^D + 3\delta^I (1-\delta^D)}{1-5(1-\delta^D)}$ \textit{when} $\gamma > \gamma_{NC}$ \textit{but by} $\frac{\delta^D}{1-5(1-\delta^D)}$ \textit{when} $\gamma < \gamma_{NC}$. Things are a bit more complicated when producers would rather stick to a noncooperative good match than look for a new supplier, as, in equilibrium, there will typically be three kinds of producers, the ones in cooperative good matches, the ones in noncooperative good matches and the other ones who have never been in a good match.} But, as the value of an innovator is higher from relationships with producers previously not in a good match relationships than from producers previously in an ongoing good match relationship, the larger is $\omega$, the larger is the incentive to innovate. Assuming that innovators do not observe the current share of producers in an ongoing good match relationship before making their investment in innovation, innovation still arises at a constant rate, but there is typically room for multiple equilibria in the cooperative case. As an equivalent to remark (2) holds, there could be one equilibrium where innovation is scarce, so that most producers have found a good match supplier and cooperation is high, and another equilibrium, where innovation is frequent, many producers are not engaged in a good match relationship and cooperation is low. Now, when innovation breaks down relationships in the Nash or contractible case but not in the cooperative case, the number of producers previously in a good match is larger in the cooperative case, which is yet another force pushing towards lower growth in the cooperative case.

**Waves of innovations in the cooperative case.** If, on the contrary, innovators were able to observe the share of producers previously not in an ongoing good match relationship, the rate of innovation (and therefore the investment levels) will not be constant overtime any more: the incentive to innovate would be larger just after an innovation has occurred. Innovations would then to occur in waves, very much in the spirit of [Stein, 1997].
5.3 Choosing the type of innovation

We revert back to the baseline equilibrium where good match suppliers resume cooperation if a producer switches supplier but only finds a bad match; and we investigate more formally the intuition that cooperation in the noncontractible case may push innovators to pursue larger innovations. To illustrate in the simplest possible framework under which circumstances this is the case, we focus on a discrete choice of two innovation regimes: regime 1 \((\gamma_1, \delta_1)\) features small but frequent innovations, while regime 2 features large but rare innovations \((\gamma_2 > \gamma_1, \delta_2 < \delta_1)\); and we investigate when regime 1 is an equilibrium, that is when it is the case that when in regime 1, an innovator has no incentive to switch to regime 2, for the contractible, Nash and cooperative cases.\(^{39}\) We denote by \((x_1, y_1)\) the equilibrium level of investment in regime 1 in the cooperative case.

Using (15), (16) and (17), the innovator has no incentive to switch to regime 2 as long as:

\[
\delta_1 \left( \omega (1 - b + b\theta) (\gamma_1 - 1) + (1 - \omega) (\gamma_1 (1 - b + b\theta) - 1)^+ \right) \\
\geq \delta_2 \left( \omega (1 - b + b\theta) (\gamma_2 - 1) + (1 - \omega) (\gamma_2 (1 - b + b\theta) - 1)^+ \right),
\]

in the contractible and Nash cases. In the cooperative case, note that if the innovator switches to regime 2, the level of investment remains \((x_1, y_1)\) except in the period where he is innovating. Indeed, in this case, the investment that good match suppliers with the outdated technology undertake (denoted \(\widetilde{y}_2\)) should reflect that in the following period, the old supplier will have access to a technology \(\gamma_2\) (and not \(\gamma_1\)) times more productive than his current technology \((\widetilde{y}_2 = \min (m, \varphi^{-1} (\gamma_2 \varphi (y_1) / \gamma_1)))\). Therefore, using (18) and (19), the innovator has no incentive to switch to regime 2 as long as:

\[
\delta_1 \left( \omega (1 - b + b\theta) (\gamma_1 \Pi (x_1) - \Pi (y_1)) + b\theta (\gamma - 1) \Pi (n)) + (1 - \omega) (\gamma_1 ((1 - b) \Pi (x_1) + b\theta \Pi (n)) - \Pi (y_1))^+ \right) \\
\geq \delta_2 \left( \omega (1 - b + b\theta) (\gamma_2 \Pi (x_1) - \Pi (\widetilde{y}_2)) + b\theta (\gamma - 1) \Pi (n)) + (1 - \omega) (\gamma_2 ((1 - b) \Pi (x_1) + b\theta \Pi (n)) - \Pi (\widetilde{y}_2))^+ \right),
\]

in the cooperative case.

For given \((\gamma_1, \delta_1, \gamma_2)\) we compute the highest innovation rate in regime 2, \(\delta_2\), that would make the innovator stick to the regime 1, in the contractible \(\left(\delta_2^C\right)\), the Nash \(\left(\delta_2^{Nash}\right)\) and the cooperative \(\left(\delta_2^{NC}\right)\) cases. The inequality \(\delta_2^C > \delta_2^{NC}\) can then be interpreted as the innovator having a higher incentive to pursue larger but scarcer innovations in the cooperative case than in the contractible case. We easily get \(\delta_2^{Nash} = \delta_2^C\), the reward from innovation is just scaled up by a factor \(\Pi (o) / \Pi (n)\) in the full contractible case, so the choice of regime by the innovator is not affected. Hence, it is only the existence of cooperation in long-term relationships that affects the relative reward from different regimes of innovations.

\(^{39}\)In a fully endogenous setting, the choice of the size and rate of innovation would depend on how cost function \(\psi\) depend on both \(\delta\) and \(\gamma\). Considering the choice between 2 regimes allows us to abstract from this issue.
Let us consider that \( \gamma_1 \in (\gamma^C, \gamma^{NC}) \). Then, in the cooperative case, the innovator does not capture the entire market under regime 1, but, as shown in Appendix F, there is a threshold \( \hat{\gamma}^{NC} \) such that for \( \gamma_2 > \hat{\gamma}^{NC} \), the innovator could capture the entire market by switching to regime 2. If \( y_1 = m \), this threshold is defined by \( \hat{\gamma}^{NC} = (1 - b)(x_1) / \Pi(m) + b \theta \Pi(n) / \Pi(m) \). \(^{40}\) We then get the following proposition.

**Proposition 5** Assume that \( \gamma_1 \in (\gamma^C, \gamma^{NC}) \), then:

(i) for \( \gamma_2 \) sufficiently large, \( \delta_{Nash}^2 > \delta_{NC}^2 \)

(ii) if \( y_1 = m \), \( \gamma_2 > \hat{\gamma}^{NC} + \omega \gamma^C - 1 \left( \hat{\gamma}^{NC} - \gamma_1 \right) / (\gamma_1 - \gamma^C) \) is a sufficient condition for \( \delta_{Nash}^2 > \delta_{NC}^2 \), it is also a necessary condition when \( x_1 = m \).

**Proof.** See Appendix F

Hence for \( \gamma_2 \) sufficiently large, the innovator has a higher incentive to switch to regime 2 in the cooperative case than in the contractible or Nash cases. The intuition is that in the cooperative case, switching allows him to capture the whole market, whereas in the two other cases, he captures the whole market even in regime 1 (the threshold is higher than \( \hat{\gamma}^{NC} \) because when \( \gamma_2 \) is slightly above \( \hat{\gamma}^{NC} \), the profits captured by the innovator from producers previously in good matches in the cooperative case are very small). The expression of the threshold when cooperation is sufficiently high (\( y_1 = m \)), \( \hat{\gamma}^{NC} + \omega \gamma^C - 1 \left( \hat{\gamma}^{NC} - \gamma_1 \right) / (\gamma_1 - \gamma^C) \), is interesting by itself. It is increasing in \( \omega \): if most producers are in a good match relationship, \( (\omega \text{ low}) \), switching in the cooperative case involves capturing most of the market size, whereas it is already captured in the contractible case. The threshold is also decreasing in \( \gamma_1 \): if \( \gamma_1 \) is close to \( \hat{\gamma}^{NC} \), switching to the regime 2 becomes more interesting in the cooperative case than in the contractible case.

Therefore, introducing cooperation in long-term relationships may lead to the surprising result, that when contractibility is weak, although the rate of innovation is reduced, the size of an incremental innovation may be larger, as long as cooperation arises.

### 5.4 Substitutability between patenting institutions and contractual institutions

In this subsection, we briefly investigate what happens if technology is not imitated after one period, for instance if firms patent their innovations and patents are well enforced, or if the technology is too complex to be soon imitated by competitors. More specifically, we assume that the innovator has a monopoly over his technology until the next innovation. Hence, at any point in time, there is a monopolist who has access to the technological frontier, while all other suppliers have access to an outdated technology \( \gamma^{-1} \) times less productive.

\(^{40}\)In general \( \gamma^{NC} \neq \gamma^{NC} \), as investment level \( x_1 \) does not adjust there, it is only when \( x_1 = m \) and the equilibrium level \( x^* \) is not decreasing in \( \gamma \) for \( \gamma > \gamma_1 \), that we have \( \gamma^{NC} = \gamma^{NC} = (1 - b + b \theta \Pi(n) / \Pi(m))^{-1} \).
The main result from this section is that in this case, the arrival of an innovation weakens cooperation in established relationships, so that cooperation in long-term relationships may not be as much a barrier to entry as before; in fact, proposition 3 can be reversed. Therefore, there is “substitutability” between patenting institutions and contractual institutions: poor enforcement of contracts reduces innovation more when patenting is poorly enforced.

To avoid a taxonomy of cases, we focus on the case where producers would rather not stick to a bad match (even the innovator), and, for the cooperative case, where they would rather look for a new supplier than stick with a noncooperative supplier (even the innovator). This will actually be the case if bad matches are sufficiently bad, but not too numerous and innovation is not too large. However, it should be clear that the intuition developed is more general.

In Appendix H, we show that in the contractible and Nash cases, a producer previously in a good match switches to the innovator if and only if:

$$\frac{(1-b)(1+\rho)}{1+\rho-(1-\delta^I)(1-\delta^I)} \left(1 - \frac{1}{\gamma}\right) + b \left(\frac{\theta - 1}{\gamma}\right) > 0. \quad (24)$$

Unsurprisingly, the threshold on the size of innovation $\gamma$ is lower than in the case where patents lasted only for one period: a producer who does not switch to the innovator ends up using the outdated technology until the next innovation arrives.

For the cooperative case, we first need to make the natural extension of the strategies played so far. We keep assuming that as long as no deviation occurred, suppliers are willing to cooperate as much as possible. We also assume that in a pair producer/supplier, if a supplier has invested less than he was supposed to, or if the producer has switched supplier and found out a new good match, cooperation in the pair ceases forever. We also consider that there is no cooperation in bad matches (so that producers do not stick to a bad match even if he is the innovator as specified above). Hence, cooperation arises only when producers and suppliers are in a good match. The level of cooperation (that is the amount of normalized investment) depends on whether the supplier is the monopolist or an outdated supplier. As in the previous section, we denote the amount of normalized investment by the innovator by $x^*$. Moreover, the level of cooperation of an outdated supplier in a good match also depends on whether the producer knows whether the innovator is a bad match or not, as this affects the outside option of the producer. We denote the amount of normalized investment by $z^*$ when the producer knows that the innovator is a bad match for him, and by $y^*$ the amount of normalized investment in the other case.

\[41\] In fact with the assumption that imitation does not occur until the following innovation, there are several equilibria in the contractible or Nash cases as well. We focus on the equilibrium where the innovator captures all the benefit from his innovation, that is the Bertrand competition equilibrium (which is the equilibrium studied in the cooperative case as well).
Appendix H shows that a producer previously in a good match switches to the innovator if and only if:

$$\frac{(1 - b) (1 + \rho)}{1 + \rho - (1 - \delta^D) (1 - \delta^I)} \left( 1 - \frac{1}{\gamma} \frac{\Pi (y^*)}{\Pi (x^*)} \right) + b \left( \theta \frac{\Pi (n)}{\Pi (x^*)} - \frac{1}{\gamma} \frac{\Pi (y^*)}{\Pi (x^*)} \right) + \frac{1}{\gamma} \frac{(1 - \delta^D) (1 - \delta^I)}{1 + \rho - (1 - \delta^D) (1 - \delta^I)} \left( \frac{\Pi (z^*)}{\Pi (x^*)} - \frac{\Pi (y^*)}{\Pi (x^*)} \right) > 0.$$  \hspace{1cm} (25)

We then get:

**Lemma 3** Consider parameter values such that a producer would rather switch to a new supplier than staying with a bad match supplier or a noncooperative good match supplier, and such in the vicinity of the parameter, there is an equilibrium where producers previously in a good match are indifferent between switching to the innovator and staying with their old supplier. The level of cooperation by outdated suppliers is higher when producers know that the innovator is a bad match for them than when they do not: $z^* \geq y^*$; and, as long as $1 + \rho > \gamma b (1 - \delta^D) (1 - \delta^I + \delta^I \gamma)$, the level of cooperation is higher with the innovator than with an outdated supplier who does not know the type of the innovator: $x^* \geq y^*$.\footnote{Technically, we make yet another assumption that an outdated supplier who is known to be a good match is a better option for the producer than a new outdated supplier. As shown in Appendix H, this does not necessarily hold because the expected value of the good match outdated supplier when the producer switches to the innovator may be sufficiently large - because of the increased cooperation in case the innovator is a bad match - that what he offers the producer to stay with him is lower than what another outdated supplier would offer to the producer. However, if this is the case, the producer would switch to the innovator even for $\gamma$ very close to 1.}

As a consequence, it is not clear anymore that producers stick to their old supplier for larger innovation size in the cooperative case than in the contractible or Nash cases. Indeed, under the hypotheses of the lemma, only the worse bad match effect (the fact that $\Pi (n)/\Pi (x^*) < 1$) would make (24) more easily satisfied than (25), having $\Pi (y^*)/\Pi (x^*) \leq 1$ and $\Pi (z^*) \geq \Pi (y^*)$ both push towards (25) being more easily satisfied than (24).

The intuition for why $z^* \geq y^*$ is easily understood. When a producer works with an outdated supplier, his outside option in the next period is better when there is a chance that the innovator is a good match than when it is known that the innovator is a bad match. The reward from cooperation for an outdated supplier is then higher in the later case, which justifies that the outdated supplier cooperates more in the later case. Knowing that outdated suppliers

\footnote{Technically, this holds for parameter values close to the ones supporting an equilibrium where producers previously in good matches are indifferent between switching or not, that is, for parameter values where the left hand side of (25) is close to 0. As we are interested in the minimal size of innovation for which the innovator captures the entire market, this is the interesting range of parameter values.}
are willing to cooperate more if the innovator turns out to be a bad match, gives the producer an additional incentive to switch in order to learn the type of the innovator.

Whether \( x^* \) is greater or smaller than \( y^* \) results from two effects. On one hand, as in the one-period patent case, there is an encouragement effect: if in the following period yet another innovation occurs, the current innovator and the current outdated supplier will have access to the same technology, so the reward from cooperation will be the same, whereas the current incentive to deviate is higher for the innovator. On the other hand, if no innovation occurs in the next period, the monopolist will still have access to a better technology than the outdated supplier. In this case, the outside option of a producer who has not tried out the innovator yet, will be to do so; whereas the outside option for a producer who has already worked with the innovator, will to start a new relationship with an outdated supplier, which is necessarily worse. As a consequence, the value captured in the following period by the innovator will be higher than the value captured by an outdated supplier, leading to a higher level of cooperation in the first place. If \( \gamma \) is sufficiently low the first effect is dominated, and \( \Pi(x^*) \geq \Pi(y^*) \), which pushes towards innovation being more easily adopted in the cooperative case than in the contractible or Nash cases. Therefore the lasting presence of a monopolist using the frontier technology can weaken cooperation in existing relationship.

Of course, it is not always the case that proposition 3 is reversed when patents last until the next innovation, or technology is easily imitated. Furthermore, in the case where suppliers stop cooperating with a producer who switches no matter whether the producer has found a new good match or not (that is in the case of subsection 5.2), the loss of cooperation effect would be another force pushing towards more stickiness in relationships in the cooperative case than in the two other cases. Rather, one should interpret the result of this subsection as showing that there may be substitutability between patenting and contractual institutions, in the sense that it is when patents are poorly enforced, that having bad contracting institutions is particularly bad. It is likely to be the case that countries with poor contracting institutions also features poor patenting enforcement...

6 Conclusion

In this paper we showed that the development of implicit contracts in a context of poor contractibility is a very poor substitute for strong institutions, particularly for economies that have reached a certain level of development. In a nutshell, our argument went as follows: incomplete contractibility leads firms to engage in cooperative long-term relationships, which can help overcoming the classic underinvestment issue associated with the lack of contractibility; however these relationships are very rigid, and slow down the process of creative destruction. Once innovation decisions are endogenized, the dynamic growth costs can be sufficiently large to overcome the static gains of cooperation. Our model predicts that with poor formal institu-
tions but informal contracting, relationships are more rigid and last longer, innovation spreads slower and is scarcer than with good formal institutions, in addition innovations are more likely to arrive in waves and may be larger when they happen. The negative effects of cooperation can be attenuated in countries far from the technological frontier, or if firms manage to avoid imitation of their innovations by competitors (for instance if they patent their innovations and patenting enforcement is of good quality). We provided first empirical evidence pointing towards more rigid relationships and towards less innovation in countries with weaker legal institutions.

In the specific context of long-term relationship and innovation, it would be interesting to add to our model the possibility for producers and suppliers to develop technologies together. Weak contractibility would typically increase the benefit from relationship-specific innovation developed in partnerships between producers and suppliers. Innovation efforts would be redirected away from general purpose innovations towards relationship-specific innovations, which would further reinforce the rigidity of relationships. Another way to extend our current analysis would be to include foreign outsourcing, issues of incomplete contractibility may be even more stringent when a firm is dealing with a supplier in a different country, as the firm may be less familiarized with the local judicial system, therefore the development of long-term relationships could be even more stringent in this context.

More generally, the idea that the cooperation in long-term relationships can lead to dynamic inefficiencies can be exploited in different contexts, varying both the reason for why firms develop long-term relationships and the source of the dynamic inefficiency. For instance, in the context of the relationship between banks and firms, the reason to engage in long-term relationships could be moral hazard or adverse selection. Another sources of dynamic inefficiency could be a too slow reallocation of resources from one sector to another in the presence of macroeconomics shocks.
References


7 Appendix 0

Table 0.1 reports the distribution of patents at the USPTO from 1992 to 2006 according to the country of the assignee, and the two measures of the quality of contractual enforcement in the corresponding countries, while table 0.2 reports the distribution of patents at the USPTO from 1992 to 2006 according to the corresponding industrial sector and the (two) measures of contractual intensity associated.
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Table 0.2
8 Appendix A

In this appendix we prove the results contained in subsection 3.4. First, we show proposition 1 and the description of the equilibrium structure; second, we derive strategies supporting an equilibrium satisfying conditions C1, C2, C3 and C4; finally, we justify condition C3 by showing the impossibility of an equilibrium featuring C1, C2 and C4 extended to bad matches, and by deriving a condition under which no pair in a bad match could renegotiate their strategies to achieve cooperation.

Part 1: Proof of proposition 1
The proof is done in 7 steps:
A) we show that investment is constant in good matches
B) we show that suppliers are engaged in Bertrand competition
C) we derive the IC constraint
D) we show that condition C4 is satisfied
F) we show the existence of a unique level of investment
G) we derive the corresponding possible ex-ante transfers

In the following we will refer to a supplier with whom a terminal deviation has occurred as a deviation supplier. Also because the strategies are independent of what happens with other producers, we focus on one producer and ignores the other producers (so we drop the index $j$).

Part 1. A: Constant joint values and investment in good matches
Condition C4 stipulates that once a good match supplier with whom no terminal deviation occurred, $k$, has been picked up by the producer, strategies must maximize the joint value under the condition that at any history belonging to $H^g_t(k)$ strategies must always be such that the joint value of the producer and the supplier is the highest possible (knowing that it must be also be the case in all subsequent histories where supplier $k$ is chosen and as long as no terminal deviation has occurred). In the same time, the strategies of the other suppliers are independent of the particular history of the game between the producer and the supplier $k$, and of the particular time period. However, the situation may be different dependent on whether the producer knows another good match or not, as this affects the outside option of the producer - it does not matter however if the producer knows more than one other good match and on whether these other good matches were deviation suppliers or not, because once the producer would have worked with the supplier $k$, all the other suppliers become deviation suppliers and their strategies are then given by condition C2-. Therefore, the situation is totally symmetric at any history in $H^g_t(k)$, at any time $t$, if the producer does not know any other good match and similarly if the producer knows at least another good match. We denote the joint value in the first case by $V^T_1$ and in the second case by $V^{TP}_1$. The joint value is the sum of the expected payoff of the producer and the chosen supplier just before the investment occurs, after the nature of the match has been revealed.
Note that the joint value $V^T_1$ must satisfy the law of motion:

$$V^T_1 = \Pi(x_t) + \frac{1 - \delta D}{1 + \rho} V^T_1,$$

where $x_t$ is the amount of investment, because $V^T_1$ is constant, $x_t$ must be constant too, equal to some level $x^*$, so that

$$V^T_1 = \frac{1 + \rho}{\rho + \delta D} \Pi(x^*).$$

Similarly the level of investment in good match relationship when the producer knows a deviation supplier is also constant in equilibrium equal to some level $x^{**}$, and we have

$$V^T_{1t} = \frac{1 + \rho}{\rho + \delta D} \Pi(x^{**}).$$

**Part 1.B: Bertrand competition.**

Suppliers make take it or leave it offer to producers, because of Part 1.A, the joint value of a pair producer - supplier is independent of the offer the supplier has made if the producer chose the supplier, this leads to Bertrand Competition.

First let us consider the case where the highest joint value is obtained from a relationship with a new supplier. Then all new suppliers should offer the entire joint value to the producer. Indeed, if one of the suppliers offers more and is chosen, his expected payoff is negative, which is not subgame perfect. If all suppliers were to offer less than the entire joint value, one supplier could offer slightly more than the best offer so far, he would be sure to capture the producer and would have a positive expected value instead of 0. Therefore, when a new relationship is the option with the highest joint value, the producer captures the entire benefit of the new relationship. Therefore, if we denote by $V^T_0$ the expected joint value of a new relationship when the producer does not know any good match supplier (before or after the ex-ante transfer has been paid, as the ex-ante transfer is just a monetary exchanged between the producer and the supplier), we get:

$$V^T_0 = V^T_0 \text{ and } V^s_0 = 0,$$

where $V^p_0$ are the value of the producer (and $V^s_0$ of the supplier), just before the ex-ante transfer is paid. Note that $V^T_0$ must then obey the law of motion:

$$V^T_0 = (1 - b) V^T_1 + b\theta \Pi(n) + b\frac{1 - \delta D}{1 + \rho} V^p_0$$

$$= (1 - b) V^T_1 + b\theta \Pi(n) + b\frac{1 - \delta D}{1 + \rho} V^T_0,$$

as in the next period the producer would again capture the joint value of the relationship. The same holds when the producer knows some deviation supplier(s), and when the joint value of
starting a new relationship is higher than the joint value of staying with a bad match. Denoting by $V_0^{T'}$ the total value of a new relationship in this case we get:

$$V_0^{p'} = V_0^{T'} \text{ and } V_0^{s'} = 0,$$

$$V_0^{T'} = (1 - b)V_1^{T'} + b\theta \Pi (n) + b\frac{1 - \delta_D}{1 + \rho} V_0^{T'}.$$

The condition that the value of a new relationship is higher than the value of a relationship with a good match with whom a terminal deviation has occurred can be expressed as:

$$\left(\frac{1 + \rho}{\rho + \delta_D} + \frac{b(1 - \theta)}{1 - b}\right) \Pi (n) < V_1^{T'}.$$

Now the same reasoning applies as soon as there are at least two suppliers offering the highest joint value. This can be the case when the producer knows at least two deviation suppliers (but none good matches with whom no terminal deviation has occurred). In this case, the value of the suppliers is always equal to 0, the producer capture the entire value of the relationship. Note that the condition under which the joint value of a new relationship is indeed smaller than the joint value of a relationship with one the deviation suppliers writes as:

$$(1 - b)V_1^{T'} + b\theta \Pi (n) + b\frac{1 - \delta_D}{1 + \rho} V_0^{T'} < V_N^T < \frac{1 + \rho}{\rho + \delta_D} \Pi (n),$$

which is equivalent to the opposite of (29).

Let us consider the case where the highest joint value is obtained with a single supplier. Then in equilibrium, the best supplier makes an ex-ante transfer that makes the producer just indifferent between choosing him or the second best supplier: if he offers less, he loses the producer in the coming period (and therefore loses the associated payoffs), if he offers more he can always just lower his offer slightly and increase his payoff. In return, the second best supplier must make an offer that leaves him with 0 expected surplus if he is chosen, otherwise, in equilibrium the offer of the first best supplier would leave the producer with what the second best supplier offers as well, so the second best supplier could make a slightly higher monetary transfer, ensuring that he is chosen and giving him a non zero expected value. In equilibrium, the strategy of the producer is then to choose the supplier making him the best offer and if two of them are making equally good offers, he should choose the one with the highest joint value. There are three scenarios following into that case.

First, consider the case where the supplier knows a good match with whom no deviation occurs and no deviation suppliers. To make the producer indifferent between switching or

\[\text{Denoting by } V_N^T \text{ the joint value when the producer works with a deviation supplier, we get:}\]

$$V_N^T = \Pi (n) + \frac{1 - \delta_D}{1 + \rho} V_0^{T'},$$

as, by assumption on the equilibrium, in the next period the producer should start a new relationship where he would capture the total value. Combining (27) and (28) we get (29).
not, it must be that the value $V_{t+1}^{p}$ of the producer and the value $V_{t+1}^{p}$ of the producer in the following period if he goes back to the supplier after having switched supplier but only to find a bad match (note that this value is conceptually different from his value on path in the next period if he stays with the same supplier all the way) follow:

$$V_{t}^{p} = (1 - b) V_{1}^{T} + b \theta \Pi (n) + b \frac{1 - \delta^{D}}{1 + \rho} V_{t+1}^{p}.$$ 

Indeed, if the producer switch, he captures the entire joint value, with probability $(1 - b)$ the new supplier is a good match and the joint value is given by $V_{1}^{T}$ (as the producer now knows a deviation supplier: the previous good match supplier), with probability $b$ the new supplier is a bad match, profits in the current period are only $n$, the continuation value of the supplier is 0, while the continuation value of the producer is the value he captures by going back to his old good match. Now defining

$$V_{t}^{p} = (1 - b) V_{1}^{T} + b \theta \Pi (n)$$

(30), using (27), we get that $V_{t+1}^{p} - V_{*} = \frac{1 + \rho}{b(1 - \delta^{D})} (V_{t}^{p} - V_{*})$, and we know that $\frac{1 + \rho}{b(1 - \delta^{D})} > 1$. Of course $V_{t+1}^{p}$ and $V_{t+2}^{p}$ (the value of the producer if he goes back to his old good match in period $t + 2$, after having switched suppliers both in period $t$ and $t + 1$, but only to find bad matches), should follow the same relation. Therefore, if $V_{t}^{p} \neq V_{*}$, $V_{t}^{p}$ should explode, which is impossible. Therefore $V_{t}^{p}$ and $V_{t}^{s}$, the value of the supplier, are constant such that:

$$V_{t}^{p} = V_{0}^{T'} = V_{*} \text{ and } V_{t}^{s} = V_{1}^{T} - V_{0}^{T'}.$$ 

(31)

Similarly if the producer knows exactly one deviation supplier and the joint value of a relationship with him is higher than the value of starting a new relationship, we get that the value of a producer $V_{N}^{p}$ and the value of a supplier $V_{N}^{s}$ must satisfy:

$$V_{N}^{p} = V_{0}^{T'} = V_{*} \text{ and } V_{N}^{s} = V_{N}^{T} - V_{0}^{T'} \text{ with } V_{N}^{T} = \frac{1 + \rho}{\rho + \delta^{D}} \Pi (n).$$ 

(32)

It is then straightforward to show that the condition under which the joint value of a relationship with a deviation supplier is lower than the value of a new relationship is exactly the reverse of (29). Note that we have now proved that whether (29) holds or not determines in all cases uniquely whether the value of a new relationship is higher or lower than the value of a relationship with a deviation supplier.

Similarly if the supplier knows a good match with whom no deviation occurs, and at least one other good match, we get that $V_{lt}^{p'}$ and $V_{lt+1}^{p'}$ defined in the same way as in the previous paragraph must follow:

$$V_{lt}^{p'} = \max \left( (1 - b) V_{1}^{T'} + b \theta \Pi (n) + b \frac{1 - \delta^{D}}{1 + \rho} V_{lt+1}^{p'}, \frac{1 - \delta^{D}}{1 + \rho} \max \left( V_{0}^{T'}, \frac{1 + \rho}{\rho + \delta^{D}} \Pi (n) \right) \right).$$
In this case the best alternative for the producer is either to switch to a new supplier or to go back to one of the deviating supplier, if it is the second option in the following period he will have the choice between starting a new relationship or still staying with the deviation supplier. Let us first consider the case where \( V_0^{T_1} > \frac{1 + \rho}{\rho + \delta D} \Pi (n) \), that is (29) holds. Then as \( V_1^{T_1} \geq V_0^{T_1} \) (a producer could always choose to start a new relationship when he has switched supplier and the supplier turned out to be a bad match instead of going back to his old good match) we get that \((1 - b) V_1^{T_1} + b \theta \Pi (n) + b \frac{1 - \delta D}{1 + \rho} V_1^{T_1} + V_1^{T_1} \geq \Pi (n) + \frac{1 - \delta D}{1 + \rho} V_0^{T_1} \). Therefore the previous argument applies and we get:

\[
V_1^{T_1} = V_0^{T_1} \quad \text{and} \quad V_1^{s_1} = V_1^{T_1} - V_0^{T_1}.
\]

Now assume on the contrary that \( V_0^{T_1} < \frac{1 + \rho}{\rho + \delta D} \Pi (n) \). Then we get

\[
V_1^{T_1} = \max \left( (1 - b) V_1^{T_1} + b \theta \Pi (n) + b \frac{1 - \delta D}{1 + \rho} V_1^{T_1}, \frac{1 + \rho}{\rho + \delta D} \Pi (n) \right),
\]

and the same holds for \( V_1^{T_2} \) and \( V_1^{T_1} \) and so on. If for all \( \tau > t \), it was the case that \((1 - b) V_1^{T_1} + b \theta \Pi (n) + b \frac{1 - \delta D}{1 + \rho} V_1^{T_1} > \frac{1 + \rho}{\rho + \delta D} \Pi (n) \), then the same logic as before would apply and \( V_1^{T_1} = V_\star \) but this would contradict the reverse of (29). Then there must be some \( \tau \), for which \((1 - b) V_1^{T_1} + b \theta \Pi (n) + b \frac{1 - \delta D}{1 + \rho} V_1^{T_1} < \frac{1 + \rho}{\rho + \delta D} \Pi (n) \), so that \( V_1^{T_1} = \frac{1 + \rho}{\rho + \delta D} \Pi (n) \), in which case the reverse of (29) directly leads to \((1 - b) V_1^{T_1} + b \theta \Pi (n) + b \frac{1 - \delta D}{1 + \rho} V_1^{T_1} < \frac{1 + \rho}{\rho + \delta D} \Pi (n) \) for all \( \tau < \tau \), so that \( V_1^{T_1} = \frac{1 + \rho}{\rho + \delta D} \Pi (n) \). Therefore we then get

\[
V_1^{T_1} = \frac{1 + \rho}{\rho + \delta D} \Pi (n) \quad \text{and} \quad V_1^{s_1} = V_1^{T_1} - \frac{1 + \rho}{\rho + \delta D} \Pi (n).
\]  

We have exhausted all possible cases and showed that Bertrand competition holds. Note that the equilibrium conditions do not uniquely pin down the value of the producer and the supplier when the producer decides to match with a supplier with whom the joint value is strictly lower than the joint value with two other suppliers. This situation however, never arises in equilibrium.

**Part 1.C, IC constraint**

Let us consider a pair producer \( j \)-supplier \( k \) who know that they are in a good match. In a SPNE, the equilibrium investment level must be such that the gain from deviating from the equilibrium investment (for the supplier) is lower than the reward from making the investment that consists of a higher continuation payoﬀ. The maximal short-run gain from deviating can be expressed as

\[
\varphi (x) = \beta R (n) - n - (\beta R (x) - x)
\]

the difference in ex-post profits when investing the Nash level and when investing the level \( x \). The continuation value is given by the value that the supplier \( k \) would capture from being in a good relationship in the following period, which is \( V_1^r \) (or \( V_1^{s_1} \)), if he does not deviate and
is 0 if he deviates except when the producer does not know any other good match supplier and the reverse of (29) holds, in which case the supplier would still capture $V_N^s$ if he deviates. Therefore the IC constraint writes as

$$
\varphi(x) \leq \frac{1 - \delta^D}{1 + \rho} V_1^s = \frac{1 - \delta^D}{1 + \rho} (V_1^T - V_*)
$$

if (29) holds and the producer does not know any other good match; as

$$
\varphi(x) \leq \frac{1 - \delta^D}{1 + \rho} V_1^{s'} = \frac{1 - \delta^D}{1 + \rho} (V_1^{T'} - V_*)
$$

if (29) holds and the producer knows another good match; as

$$
\varphi(x) \leq \frac{1 - \delta^D}{1 + \rho} (V_1^s - V_N^s) = \frac{1 - \delta^D}{1 + \rho} \left( V_1^T - \frac{1 + \rho}{\rho + \delta^D} \Pi(n) \right)
$$

if the reverse of (29) holds and the producer does not know any other good match; and as

$$
\varphi(x) \leq \frac{1 - \delta^D}{1 + \rho} V_1^{s'} = \frac{1 - \delta^D}{1 + \rho} \left( V_1^{T'} - \frac{1 + \rho}{\rho + \delta^D} \Pi(n) \right)
$$

if the reverse of (29) holds and the producer knows at least another good match.

Now condition C4 stipulates that the strategies must be such that the joint value is maximized amongst the set of strategies for which the joint value is maximized in all subsequent histories in $H_{t+\tau}^D(k)$. As we already said, if such an equilibrium exist, the investment level in a good match relationship must be constant. Knowing that $V_1^T$ and $V_1^{T'}$ are the discounted value of the joint total profits derived from the investment undertaken by the supplier (while $V_*$ depends only on the investment level of the other suppliers: $V_* = \frac{(1-b)^{1+\rho}\Pi(x^*)+b\Pi(n)}{1-b^{1+\rho}}$),

the investment level should then be the highest element in $(n,m)$ such that

$$
\varphi(x) \leq \frac{1 - \delta^D}{1 + \rho} \left( \frac{1 + \rho}{\rho + \delta^D} \Pi(x) - V \right), \quad (34)
$$

where $V = \max \left(V_*, \frac{1+\rho}{\rho+\delta^D} \Pi(n) \right)$, we denote that element $\tilde{x}$ (which therefore depends on the equilibrium level of investment through its dependence on $V_*$).

**Part 1D. The equilibrium satisfies C4**

Now we need to check that the equilibrium actually exists, that is we need to check that there are no strategies still satisfying C4 in the next period, and such that the entire profile is still a SPNE, which would improve the joint value of the relationship. Note, that no matter what these alternative strategies could be, as long as they form a SPNE with the (given) strategies played by the other supplier, $V_*$ remains the same constant. Further note that if the producer randomly switches suppliers, it would not increase the incentive to invest of the supplier and cannot increase the joint value of the relationship; moreover, if the supplier
underinvests, it decreases the joint profit in the period of deviation, and the punishment specified by C2 is the harshest one. The only way an alternative strategy could do better then is by choosing an alternative profile of investment, however any alternative profile must still satisfy C4 and be SPNE, and so investment is bound to be given by $\bar{x}$ in future periods. Yet, the highest element in $(n, m]$ satisfying $\varphi(x) \leq \frac{1-\delta^D}{1+\rho} \left( \frac{1+\rho}{\rho+\delta^D} \Pi(m) - V \right)$ is precisely $\bar{x}$: if the constraint is not binding, $\varphi(m) \leq \frac{1-\delta^D}{1+\rho} \left( \frac{1+\rho}{\rho+\delta^D} \Pi(m) - V \right) \leq \frac{1-\delta^D}{1+\rho} \left( \frac{1+\rho}{\rho+\delta^D} \Pi(m) - V \right)$, so that $\bar{x} = m = x$, otherwise the constraint is binding and $\varphi(x) = \frac{1-\delta^D}{1+\rho} \left( \frac{1+\rho}{\rho+\delta^D} \Pi(x) - V \right) = \varphi(\bar{x})$, which implies $x = \bar{x}$ as $\varphi$ is strictly increasing on $(n, m]$. Therefore the strategies describe so far satisfy C4.

**Part 1E. Equilibrium level of investment**

Note that (34) does not change whether the producer knows another good match or not, therefore, in equilibrium, we get $V_1^T = V_1^{T'}$ and $x^* = x^*$. $x^*$ must then satisfy either:

$$x^* = m \quad \text{and} \quad \varphi(m) \leq \frac{1-\delta^D}{1+\rho} \left( \frac{1+\rho}{\rho+\delta^D} \Pi(m) - \max \left( \frac{(1-b) \frac{1+\rho}{\rho+\delta^D} \Pi(m) + b \theta \Pi(n)}{1-b \frac{1-\delta^D}{1+\rho}}, \frac{1+\rho}{\rho+\delta^D} \Pi(n) \right) \right),$$

or

$$x^* \in (n, m) \quad \text{and} \quad \varphi(x^*) = \frac{1-\delta^D}{1+\rho} \left( \frac{1+\rho}{\rho+\delta^D} \Pi(x^*) - \max \left( \frac{(1-b) \frac{1+\rho}{\rho+\delta^D} \Pi(x^*) + b \theta \Pi(n)}{1-b \frac{1-\delta^D}{1+\rho}}, \frac{1+\rho}{\rho+\delta^D} \Pi(n) \right) \right).$$

Let us define

$$g(x) \equiv \varphi(x) - \frac{1-\delta^D}{1+\rho - b (1-\delta^D)} b (\Pi(x) - \theta \Pi(n)) + \max \left( \frac{(1-\delta^D) (1+\rho) (1-b)}{(1+\rho - b (1-\delta^D)) (\rho + \delta^D)} \left( \frac{1+b (\rho + \delta^D) (1-\theta)}{(1-b) (1+\rho)} \Pi(n) - \Pi(x) \right) \right), 0 \right).$$

Then, all the possible equilibrium levels of investment need to satisfy either $x = m$ and $g(m) \leq 0$ or $x \in (n, m)$ and $g(x) = 0$. Now, $g(n) \leq 0$, and $g$ is convex as $\varphi(x)$ is convex, $\Pi(x)$ concave and max of a convex function is itself convex, so that if $g(m) \leq 0$ there is no solution to $g(x) = 0$ in $(n, m)$, and if $g(m) > 0$, there is a unique solution to $g(x) = 0$.

Therefore, there is a unique candidate equilibrium level of investment. All we need to do now is to exhibit strategies sustaining such a SPNE. In case of a terminal deviation, the producer would (no matter the circumstances) look for a new supplier rather than working with one of the deviation suppliers if (29) holds that is if:

$$\left( 1 + \frac{b (\rho + \delta^D) (1-\theta)}{(1-b) (1+\rho)} \right) \Pi(n) < \Pi(x^*) \tag{35}$$

if the reverse inequality holds, he will not, and if there is equality, he can do so with any probability.
**Part 1 G: Corresponding ex-ante transfers**

Before describing the equilibrium strategies in the next section, however, we compute the ex-ante transfers corresponding to the equilibrium.\(^{45}\)

If (35) holds, or if the reverse of (35) holds but the producer does not know any other good match, the value of a supplier in a good match \(V_1^s\) (or \(V_1^{st}\)) is given by (31), that is

\[
V_1^s = \frac{1 + \rho}{\rho + \delta^D} \Pi(x^*) - \frac{(1 - b) \frac{1 + \rho}{\rho + \delta^D} \Pi(x^*) + b \theta \Pi(n)}{1 - b \frac{1 - \delta^D}{1 + \rho}} = \frac{(1 + \rho) b (\Pi(x^*) - \theta \Pi(n))}{1 + \rho - b (1 - \delta^D)},
\]

so that the ex-ante transfer \(t_1\) paid must be such that:

\[
-t_1 + \beta R(x^*) - x^* + 1 - \delta^D \frac{V_1^s}{1 + \rho} = V_1^s = \frac{(1 + \rho) b (\Pi(x^*) - \theta \Pi(n))}{1 + \rho - b (1 - \delta^D)} \Rightarrow
\]

\[
t_1 = \beta R(x^*) - x^* - \left( \frac{\rho + \delta^D}{1 + \rho - b (1 - \delta^D)} b (\Pi(x^*) - \theta \Pi(n)) \right).
\]

When the producer does not know any good match, or when the producer knows some good matches but (35) holds, \(V_0^s = 0\) and in case of a good match the value of the supplier would be \(V_1^s\). Therefore the ex-ante monetary transfer \(t_0\) offered by suppliers who have never worked with the final good producer must be:

\[
t_0 = (1 - b) \left( \beta R(x^*) - x^* + 1 - \delta^D \frac{V_1^s}{1 + \rho} \right) + b \theta (\beta R(n) - n)
\]

\[
= (1 - b) \left( \beta R(x^*) - x^* + \frac{1 - \delta^D}{1 + \rho - b (1 - \delta^D)} b (\Pi(x^*) - \theta \Pi(n)) \right) + b \theta (\beta R(n) - n).
\]

When the reverse of (35) holds, and the producer knows a deviation supplier, using (33), the value of a supplier in a good match (with whom no deviation has occurred) is given by:

\[
V_1^{st} = \frac{1 + \rho}{\rho + \delta^D} (\Pi(x^*) - \Pi(n)),
\]

so that the ex-ante transfer \(t_1'\) is given by

\[
-t_1' + \beta R(x^*) - x^* + 1 - \delta^D \frac{V_1^{st}}{1 + \rho} = V_1^{st} = \frac{1 + \rho}{\rho + \delta^D} (\Pi(x^*) - \Pi(n)) \Rightarrow
\]

\[
t_1' = \beta R(x^*) - x^* - (\Pi(x^*) - \Pi(n)).
\]

Now, still when the reverse of (35) holds, if the producer knows exactly one good match supplier, the value of suppliers who have never worked with the producer must be null if they

\(^{45}\)We describe the ex-ante transfer when either (35) or its reverse hold, when there is actually equality in (35), the transfers must (and can) satisfy both sets of constraints.
are chosen - \( V_0'' = 0 \), if these suppliers turn out to be a good match, their value will be given by \( V_1'' \), so the ex-ante transfer they offer \( t'_0 \), must be:

\[
t'_0 = (1 - b) \left( \beta R(x^*) - x^* + \frac{1 - \delta D}{1 + \rho} V_1'' \right) + b \theta (\beta R(n) - n)
\]

\[
= (1 - b) \left( \beta R(x^*) - x^* + \frac{1 - \delta D}{\rho + \delta D} (\Pi(x^*) - \Pi(n)) \right) + b \theta (\beta R(n) - n).
\]

If the producer knows at least two good matches (with whom a deviation has occurred or not), the value of the relationship with a new supplier is no longer the second highest one, so the only restriction on the transfer is that it has to be at least as small as \( t'_0 \). However in condition C1, we required that the strategies do not depend on whether the producer knew more than one deviation supplier or not, so we have to fix the transfer at \( t'_0 \) there too.

If the reverse of (35) holds, the value of a deviation supplier when he is in a relationship with a producer who does not know any other good match is given by (using (32)):

\[
V_n^s = \frac{1 + \rho}{\rho + \delta D} \Pi(n) - V_0^T
\]

\[
= \frac{(1 + \rho)^2 (1 - b)}{(\rho + \delta D) (1 + \rho - b (1 - \delta D))} \left( \frac{b (1 - \theta) (\rho + \delta D)}{(1 + \rho) (1 - b)} \right) \Pi(n) - \Pi(x^*)
\]

therefore the ex-ante transfer he offers must be:

\[
t_{n2} = \beta R(n) - n - \frac{\delta D + \rho V_n^s}{1 + \rho}
\]

\[
= \beta R(n) - n - \frac{(1 + \rho) (1 - b)}{(1 + \rho - b (1 - \delta D))} \left( \frac{b (1 - \theta) (\rho + \delta D)}{(1 + \rho) (1 - b)} \right) \Pi(n) - \Pi(x^*)
\]

If the reverse of (35) holds and the producer knows several deviation supplier, the joint value in a relationship with one of them is at least the second highest value, so the value of one of these deviation suppliers must be 0, therefore they offer an ex-ante transfer \( t_{n1} \):

\[
t_{n1} = (\beta R(n) - n),
\]

However, if (??) is not satisfied, the value of a relationship with a noncooperative good match is lower than the value of a new relationship so any transfer equal or lower than \( t_{n1} \) is possible in equilibrium.

Finally the value of a relationship with a bad match supplier is never the highest or second highest value, so bad match supplier should offer ex-ante transfer that are lower or equal to:

\[
t_{nb} = \theta (\beta R(n) - n).
\]

This achieves the description of equilibria satisfying C1, C2, C3 and C4

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Part 2: Strategies to sustained a SPNE satisfying C1, C2, C3 and C4

We call "new" suppliers, suppliers with whom the producer has never worked before, and we keep calling "deviation" supplier, a good match supplier with whom a terminal deviation has occurred.

Part 2.A: Case 1: $x^*$ satisfies (35).

A new producer starts in phase $P0$.

Phase $P0$: the producer is not currently in a good match relationship.

We call

1. New suppliers offer $t_0$, those who know they are bad matches offer $t_{nb}$, deviation suppliers offer $t_{n1}$.

2. The producer chooses a supplier amongst the set $S$ of those with whom his expected value is the highest possible, choosing preferably amongst the new suppliers.\footnote{On equilibrium path the set $S$ consists precisely of the set of new suppliers, this formulation allows to consider off path cases where a bad match or a good match with whom a deviation has occurred offers such a high ex-ante transfer that the producer should work with one of them for one period.}

3. If the producer picks up a new supplier, and the supplier turns out to be a good match, the supplier invests $x$. In any other cases a bad match supplier invests $\theta n$, and a deviation supplier invests $n$.

4. If the producer has worked with a new supplier, who turned out to be a good matched and invested at least $x^*$, move to phase $P1$; otherwise stay in phase $P0$.

Phase $P1$: the producer is currently in a good match relationship.

We call "ongoing" supplier the last good match supplier who has worked with the producer, and we call "deviation" supplier any other good match supplier who has worked with the producer (so that a deviation must have occurred).

1. The old supplier offers $t_1$, new suppliers offer $t_0$, those who know they are bad matches offer $t_{nb}$, deviation suppliers offer $t_{n1}$.

2. The producer chooses a supplier amongst the set $S$ of those with whom his expected value is the highest possible, choosing preferably the ongoing supplier, and if he is not in the set $S$, choosing preferably a new supplier.

3. If the producer picked up the ongoing supplier or if he picked a new supplier who turns out to be a good match, the supplier invests $x^*$. Otherwise the supplier invests $\theta n$ if he is a bad match and $n$ if he is a deviation supplier.
4. If the producer has kept working with the ongoing supplier, and the ongoing supplier has
invested at least $x^*$, or if the producer has worked with a new supplier, who turned out
to be a good matched and has invested at least $x^*$, or if the supplier has worked with a
bad match, stay in phase $P_1$; otherwise move to phase $P_0$.\footnote{This includes the cases where the supplier deviated and where the producer worked with a good match who is neither the ongoing good match or a new good match.}

Thanks to the discussion above it is direct that this is SPNE. On path the following will
occur: a producer switches suppliers till he find a good match, once he has found a good match,
the good match invests $x^*$, and the producer stays with him. If a producer deviates but ends
up with a bad match the supplier forgives him, if the producer goes back to a previous good
match or if the supplier deviates invests less than he ought to, the producer moves back to the
initial phase.

**Part 2B Case 2: $x^*$ does not satisfy (35)**

A new producer starts in phase $P_0$

Phase $P_0$: the producer has never found a good match supplier

1. New suppliers offer $t_0$, those who know they are bad matches offer $t_{nb}$.

2. The producer chooses a supplier amongst the set $S$ of those with whom his expected
value is the highest possible, choosing preferably amongst new suppliers.

3. If the supplier turns out to be a good match, the supplier invests $x^*$. If the supplier is a
bad match he invests $\theta n$.

4. If the producer has worked with a good match supplier who invested at least $x^*$ move to
phase $P_1$, if he has worked with a good match who invested less than $x^*$ move to phase
$P_2$, otherwise stay in phase $P_0$.

Phase $P_1$: the producer is in an ongoing relationship and has never worked with any other
good match supplier before

We call ongoing supplier the supplier with whom the producer was working in the previous
period.

1. The ongoing supplier offers $t_1$, new suppliers offer $t_0$, those who know they are bad
matches offer $t_{nb}$.

2. The producer chooses a supplier amongst the set $S$ of those with whom his expected
value is the highest possible, choosing preferably the ongoing supplier, and if he is not
in the set $S$, choosing preferably a new supplier.
3. If the supplier is a good match, the supplier invests $x^*$. If the supplier is a bad match, the supplier invests $\theta n$.

4. If the producer has kept working with the ongoing supplier, and the ongoing supplier has invested at least $x^*$, or if the producer has worked with a bad match supplier, stay in phase $P_1$; if the producer has kept working with the ongoing supplier, but the ongoing supplier has invested less than $x^*$, move to phase $P_2$; if the producer has switched to a new supplier, the new supplier turned out to be a good match and has invested at least $x^*$, move to phase $P_3$; if the producer has switched to a new supplier who turned out to be a good match, but invested less than $x^*$, move to phase $P_4$.

Phase $P_2$: the producer knows exactly one good match supplier, but a deviation occurred with him.

We call "deviation" supplier the supplier who is a good match but with whom a deviation has occurred.

1. The deviation supplier offers $t_{n2}$, new suppliers offer $t'_0$, those who know they are bad matches offer $t_{nb}$.

2. The producer chooses a supplier amongst the set $S$ of those with whom his expected value is the highest possible, choosing preferably the deviation supplier, and if he is not in the set $S$, choosing preferably a new supplier.

3. If the deviation supplier was chosen, he invests $n$; if the supplier is a new supplier and a good match he invests $x^*$; if the supplier is a bad match he invests $\theta n$.

4. If the producer has switched to a new supplier, the new supplier turned out to be a good match and invested at least $x^*$, move to phase $P_3$; if the producer has switched to a new supplier who turned out to be a good match but invested less than $x^*$, move to phase $P_4$; in any other cases stay in phase $P_2$.

Phase $P_3$: the producer is in an ongoing relationship but he knows other good match producers with whom a deviation has occurred.

We call ongoing supplier the last good match supplier with whom the producer has been working; and "deviation" suppliers the other good match suppliers.

1. The ongoing supplier offers $t'_1$, deviation suppliers offer $t_{n1}$, new suppliers offer $t'_0$, those who know they are bad matches offer $t_{nb}$.

2. The producer chooses a supplier amongst the set $S$ of those with whom his expected value is the highest possible, choosing preferably the ongoing supplier; if he is not in the set $S$, choosing preferably a deviation supplier; if neither of them are in the set, choosing preferably a new supplier.

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3. If the supplier is either the ongoing supplier or a new supplier who turned out to be a 
good match, he invests $x^*$; if the supplier is a deviation supplier, he invests $n$; if he is a 
bad match, he invests $\theta n$.

4. If the producer has kept working with the ongoing supplier, and the ongoing supplier has 
invested at least $x^*$, or if the producer has switched to a new supplier, the new supplier 
turned out to be a good match and invested at least $x^*$, or if the producer has worked 
with a bad match supplier, stay in phase $P3$; otherwise move to phase $P4$.

Phase $P4$: the producer knows at least two good match suppliers with whom a deviation 
has occurred and is not in an ongoing relationship.

We call "deviation" suppliers the good match suppliers, the producer has been working 
with.

1. Deviation suppliers offer $t_{n1}$, new suppliers offer $t'_0$, those who know they are bad matches 
offer $t_{nb}$.

2. The producer chooses a supplier amongst the set $S$ of those with whom his expected 
value is the highest possible, choosing preferably a deviation supplier; if neither of them 
are in the set, choosing preferably a new supplier.

3. If the supplier is a deviation supplier, he invests $n$; if the supplier is a new supplier who 
turned out to be a good match, he invests $x^*$; if he is a bad match, he invests $\theta n$.

4. If the producer has worked with a new supplier who turned out to be a good match and 
invested at least $x^*$, move to phase $P3$, otherwise stay in phase $P4$.

Once again it should be pretty obvious that these strategies give rise to a SPNE satisfying 
$C1$, $C2$ and $C3$. On path the following will occur: a producer switches suppliers till he find a 
good match, once he has found a good match, the good match invests $x^*$, and the producer 
stays with him. If the producer deviates but finds a bad match he is forgiven, if the producer 
deviates and finds a good match, the new good match cooperates. If the supplier deviates and 
invests less than $x^*$, the producer keeps working with him but the supplier only invest $n$ from 
then on.

$Part 3: Justifying the choice of this particular SPNE$

$Part 2.A$: There is no symmetric SPNE satisfying $C1$, $C2$ and $C4$ also after histories 
belonging to $H^*_b (j,k)$.

If that were the case, investment in bad matches would also be given by a constant $x_b > n$, 
and the supplier would have to be kept with positive probability. To get cooperation in good 
and bad matches, both the value of a relationship with a good match and the value of a
relationship with a bad match need to be strictly greater than the value of a new relationship, which is a weighted average of the two of them: hence there is a contradiction.

**Part 2.B:** Deriving condition under which no cooperation would be possible in a pair producer - supplier in a bad match.

Let us assume that a pair producer - supplier in a bad match cooperates. If this is possible, it is possible to cooperate with a weakly increasing amount of cooperation, let us call $x_b$, the limit level of investment. Then $x_b$ should satisfy

$$
\theta \varphi (x_b) \leq \frac{1 - \delta^D}{\rho + \delta^D} \theta \Pi (x_b) - V_0^T \Rightarrow
$$

$$
\theta \varphi (x_b) \leq \frac{1 - \delta^D}{(1 + \rho - b (1 - \delta^D)) (\rho + \delta^D)} \left( (1 + \rho) (1 - b) (\theta \Pi (x_b) - \Pi (x)) + b (\rho + \delta^D) \theta (\Pi (x_b) - \Pi (n)) \right).
$$

If this has no solution in $(n, m]$, then no pair in a bad match would like to renegotiate their strategies and to start cooperating.