Abstract—This paper proposes a new method for transient stability analysis for electric power systems. Different from existing methods, a minimization problem with boundary values is formulated for obtaining critical conditions for transient stability, where a new numerical integration method is developed by modifying the trapezoidal formula to solve effectively the boundary value problem. The proposed method is to compute directly a trajectory on the stability boundary, which is referred to as critical trajectory in this paper. The critical trajectory to be obtained is the trajectory that starts from a point on a fault-on trajectory and reaches a critical point such as an unstable equilibrium point (UEP), or more exactly, controlling UEP (CUEP). The solution of the minimization problem provides critical clearing time (CCT) and exit point simultaneously.

Index Terms—Electric power system, Transient stability, Transient energy function, Unstable equilibrium.

I. INTRODUCTION

Transient stability analysis plays an important role for maintaining security of power system operation. The analysis is mainly performed through numerical simulations, where numerical integration is carried out step by step from an initial value to obtain dynamic response to disturbances. In general, such a numerical simulation method is effective since it easily takes into account various dynamic models for complex power systems as well as various time sequences of events. Furthermore, the method is useful in analyzing various kinds of complex nonlinear phenomena such as in [1-3]. However, the numerical simulation is usually time consuming, and therefore, it is not necessarily suited for real time stability assessment.

An alternative approach, called transient energy function methods [4-16], assesses system stability based on the transient energy. Those methods provide fast and efficient stability assessment for a number of disturbances. Although they are practically useful, a common disadvantage is concerned with the accuracy of stability judgment. A major limitation is that they cannot deal with detailed models for power systems since the transient energy functions are available only for limited types of power system models. Another problem is that the most of the methods require the evaluation of critical energy, which affects considerably the accuracy of stability assessment. The critical energy is not necessarily easily calculated.

Among various transient energy function methods, the Boundary Controlling Unstable (BCU) equilibrium point method seems to be a promising method in the sense that it has a theoretical background for the evaluation of the critical energy [10-16]. The method evaluates the critical energy at CUEP. Improved techniques have been proposed in [12, 13], while there are discussions on the underlying assumption for the BCU method [14-16].

This paper proposes a new method for transient stability analysis. A preliminary investigation has been performed by the authors in [17,18] . In order to describe the proposed method, typical dynamic behaviors of a power system are given in Fig. 1, where a single machine case without damping is presented as an example. Three kinds of trajectories are given in phase plane starting at different points on a fault-on trajectory “1”. Trajectory “2” is for a stable case where the fault is cleared early enough and it oscillates around a stable equilibrium point (SEP). Trajectory “4” corresponds to an unstable case, where the fault clearing is too late. Trajectory “3” corresponds to a critical case for stability and is referred to as critical trajectory in this paper.

The critical trajectory starts from a point on a fault-on trajectory and reaches CUEP. It is generally difficult to compute the critical trajectory by means of conventional numerical simulations. In this paper, the problem is newly formulated as a minimization problem for computing the critical trajectory. Some important characteristics may be

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denoted as follows: (1) The proposed method at least in theory can compute directly exact CCT for general power system models. There have been no such computation methods so far. (2) A minimization problem is formulated for transient stability problem. (3) The formulation for obtaining dynamic behavior is not based on an initial value problem but based on boundary value problem. (4) A new numerical integration (simulation) method is developed for the boundary value problem utilized in the formulation.

II. MATHEMATICAL BACKGROUND

Transient stability problem for a event disturbance may be expressed as follows: Initially, a power system is operating at a stable operating point, say $x_{pv}$, when a fault occurs at time $t = 0$. Then, the system is governed by the fault-on dynamics during the fault $[0, \tau]$ as follows:

$$ \dot{x} = f_{s}(x), \ 0 \leq t \leq \tau, \ x(0) = x_{pv} $$ (1)

where $x \in \mathbb{R}^n$, $t \in \mathbb{R}$, $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$

The solution curve of (1) is called fault-on trajectory and is expressed in this paper by:

$$ x(t) = X_{s}(t; x_{pv}), \ 0 \leq t \leq \tau $$ (2)

where $X_{s}(\cdot; x_{pv}) : \mathbb{R} \rightarrow \mathbb{R}^n$

The fault is cleared at time $\tau$. The system is governed by the post-fault dynamics expressed by the following nonlinear equation:

$$ \dot{x} = f(x), \ \tau \leq t < \infty, \ f: \mathbb{R}^n \rightarrow \mathbb{R}^n $$ (3)

The solution curves of (3) are called post-fault trajectory, represented by

$$ x(t) = X(t; x^{0}), \ \tau \leq t < \infty, \ X(\cdot; x^{0}) : \mathbb{R} \rightarrow \mathbb{R}^n $$ (4)

Note that initial point $x^{0}$ is a point on the fault-on trajectory at time $\tau$, fault clearing time.

$$ x^{0} = X_{s}(\tau; x_{pv}) $$ (5)

In general, there exists stable and unstable equilibrium points (SEP and UEP) of post-fault system (3), denoted respectively as $x_{s}$ and $x_{u}$, which satisfy

$$ 0 = f(x) $$ (6)

Stability region around $x_{s}$ is expressed as $A_{s}(x_{s})$, in which all the solutions converge to $x_{s}$ as follows:

$$ A_{s}(x_{s}) = \left\{ x \in \mathbb{R}^n \mid \lim_{t \rightarrow \tau} X(t; x) = x_{s} \right\} $$ (7)

Boundary of the stability region is denoted as $\partial A_{s}(x_{s})$. A typical situation is a case where fault-on trajectory (2) starts at $t=0$ from inside $A_{s}(x_{s})$, goes apart from $x_{s}$, and then intersects $\partial A_{s}(x_{s})$ at $t = t_{c}$, critical clearing time (CCT). This implies that the system is stable if the fault is cleared before $t_{c}$ but unstable after $t_{c}$. The post-fault trajectory corresponding to the critical case is referred to as critical trajectory in this paper, which is defined as follows:

$$ x(t) = X(t; x^{0}), \ t_{c} \leq t < \infty $$ (8)

where $x^{0} = X_{s}(t_{c}; x_{pv}) \in \partial A_{s}(x^{0})$

This paper aims at obtaining directly $t_{c}$, or CCT, together with the critical trajectory. For this purpose, we pay attention to an interesting characteristic concerned with stability boundary for certain systems, shown by H. D. Chiang et al in [11] as follows.

Corollary 4-3 in [11]: “If there exist an energy function for system (3) which has an asymptotically stable equilibrium point $x_{c}$ (but not globally asymptotically stable), then the stability boundary $\partial A_{s}(x_{c})$ is contained in the set which is the union of the stable manifolds of the UEP’s on the stability boundary $\partial A_{s}(x_{c})$.” Since any point on the stable manifold converges to the corresponding UEP, the corollary implies that for a system having an energy function, the critical trajectory of (8) must converge to one of the UEP’s, say CUEP, $x_{a}$. That is,

$$ \lim_{t \rightarrow \infty} X(t; x^{0}) = x_{a} $$ (9)

III. PROBLEM FORMULATION

Letting a solution of equation (3) at time $\tau$ be denoted as $\dot{x}^{\tau}$, the following equation holds using the trapezoidal formula.

$$ x^{t_{c} + 1} - x^{t_{c}} = \frac{1}{2} x^{t_{c} + 1} + x^{t_{c}} (t_{c} + 1 - t_{c}) $$ (10)

where,

$$ \dot{x}^{t_{c}} = f(x^{t_{c}}) $$

In this paper, superscript $k$ is used for state transition number with respect to time.

Now we pay attention to the critical trajectory, where a system fault is cleared at CCT and then the state variables converge to CUEP with infinite time, satisfying condition (9). The critical trajectory, where two boundary points, $x^{*}$ and $x_{s}$, represent the initial point at CCT and CUEP respectively. A difficulty in obtaining the critical trajectory is that infinite time is taken to reach CUEP as seen in condition (9). To avoid the problem, the distance between the two points in (10) is defined as:

$$ \varepsilon = \left| x^{t_{c} + 1} - x^{t_{c}} \right| = \frac{1}{2} \left| x^{t_{c} + 1} + x^{t_{c}} \right| (t_{c} + 1 - t_{c}) $$ (11)

Then, the time duration is replaced with the distance as follows:

$$ t_{c} + 1 - t_{c} = \frac{2}{\left| x^{t_{c} + 1} - x^{t_{c}} \right|} \varepsilon $$ (12)

Equation (12) is substituted into (10) to obtain the following new integration form.

$$ x^{t_{c} + 1} - x^{t_{c}} = \frac{x^{t_{c} + 1} - x^{t_{c}}}{\left| x^{t_{c} + 1} - x^{t_{c}} \right|} \varepsilon = 0 $$ (13)

The conventional numerical integration with respect to time is now transformed into that with distance in (13). This transformation makes it possible to represent the critical trajectory by finite points with a same distance, $\varepsilon$. Then, the problem for obtaining the critical trajectory for system (3) is formulated as follows:

$$ \min_{\varepsilon \in \mathbb{R}} \left\{ \sum_{k=0}^{m+1} \mu^{k} \varepsilon^{k} + \mu^{m+1} W \mu^{m+1} \right\} $$ (14)

where, $x^{k} \in \mathbb{R}^{n}, (k = 0, \ldots, m + 1), \varepsilon \in \mathbb{R}, \tau \in \mathbb{R}$,
\[ \mu^k = x^{k+1} - x^k - \frac{x^{k+1} + x^k}{2} \]  
\[ x^1 = f(x^0) \]  
with boundary conditions:
\[ x^0 = X_j(t,x_{pv}) \]  
\[ \mu^{n+1} = x^{n+1} - x_n \] with \( f(x_n) = 0 \)

In the above equations, \( \mu \) is ideally zero, implying a numerical error due to the trapezoidal approximation of (15). Equation (17) is identical with (5), implying that the initial point is on a fault-on trajectory as a function of fault clearing time. \( r \). We assume that the fault-on trajectory is obtained numerically in advance as a function of \( r \). Equation (18) is the other boundary condition equivalent to (9), where \( x_n \) is CUEP that satisfies equilibrium equation \( f(x_n) = 0 \). It is noted that CUEP, \( x_n \), may be determined separately in advance by other methods such as the BCU method.

\( W \) is weighting matrix to specify selectively the boundary conditions of state variables. Selection of \( W \) will be discussed in section V.

IV. APPLICATION TO BCU METHOD

The formulation proposed in the previous section is applied to the BCU method [10, 11, 12], where a simple, fictitious system called the gradient system is used to find CUEP.

A distinctive characteristic of the BCU method as a transient energy function method exactly lies in computing CUEP using the gradient system and in evaluating the critical energy at the CUEP; the other procedures are almost in common with the other transient energy function methods.

In the following, the gradient system is used for (16) in the proposed minimization problem to develop a new alternative approach for finding CUEP. Then, we will check if the same solution of CUEP is obtained, compared with the shadowing method. Although there are discussions on the underlying assumption for the BCU method [14-16], we will not discuss those issues but will focus on the verification of the effectiveness of the proposed method. The purpose of the examination in this section is to check if the same solutions of CUEP are obtained by the shadowing method and the proposed method.

A. Power System Model

We consider \( n \)-machine power system model as follows:
\[ M_i \theta_i = P_{\omega_i} - P_i(\theta) \]  
\[ \dot{\theta}_i = \omega_i - \dot{x}_i, \quad i = 1, \cdots, n \]  
where,
\[ P_i(\theta) = \sum_{j=1}^{n} V_j E_j E_n \sin(\theta - \theta_j + \alpha_j) \]

The energy function corresponding to this system is given as the sum of kinetic and potential energy terms as follows:
\[ V = \frac{1}{2} \sum_{i=1}^{n} M_i \dot{\theta}_i^2 + \sum_{i=1}^{n} \int P_i(\delta) - P_{\omega_i} + \frac{M_i}{M_r} P_{\omega_i}(\delta) d\delta = V_i(\delta) + V_r(\delta) \]

where,
\[ \delta_i = \theta_i - \frac{1}{M_r} \sum_{j=1}^{n} M_j \theta_j, \quad \omega_i = \omega_i - \frac{1}{M_r} \sum_{j=1}^{n} M_j \omega_j \]
\[ P_{\omega_i}(\delta) = \sum_{j=1}^{n} (P_{\omega_j} - P_{\omega_i}(\delta)), \quad M_r = \sum_{i=1}^{n} M_i \]

In order to compute CUEP, the following system called the gradient system, the gradient of the potential energy, has been proposed to be used in the BCU method [10, 11].
\[ \delta = \frac{\partial V_i(\delta)}{\partial \delta} \]

B. A Method for Obtaining CUEP

The system (3) to be analyzed is now given by the gradient system (21), which is equivalently expressed as follows:
\[ \dot{\delta}_i = P_{\omega_i} - P_i(\delta) - \frac{M_i}{M_r} P_{\omega_i}(\delta), \quad i = 1, \cdots, n \]

The state vector in (3) is:
\[ x = [\delta_1, \delta_2, \cdots, \delta_n]^T \]
For an initial point corresponding to conditions (17), we use a fixed point as follows.
\[ \delta^e = \delta^{exit} \]
where \( \delta^{exit} \) is the exit point detected as the first local maxima of the potential energy, \( V_r \), on the fault-on trajectory. Note that the trajectory starting at (24) is uniquely defined as the critical trajectory, which is identical with that defined by the BCU method, implying that the trajectory converges to an unique UEP, that is CUEP. For the condition for UEP, we use the following form of the power flow equations, corresponding to (18).
\[ P_{\omega_i} - P_i(\delta^e) - \frac{M_i}{M_r} P_{\omega_i}(\delta^e) = 0, \quad i = 1, \cdots, n \]

Thus, the minimization problem (14)-(18) is defined. Then, we will use the Newton’s method to solve the condition of minimizing the least square error for the set of equations. The computational procedure is given as follows:

a. Compute fault-on trajectory by the conventional simulation method and obtain the exit point, (24).

b. Solve the minimization problem (14)-(18) to obtain CUEP, \( \delta^e = [\delta_1^e, \delta_2^e, \cdots, \delta_n^e]^T \).

C. Numerical Examinations

Numerical examinations are carried out using 3-machine 9-bus system [19] and IEEE 6-machine 30-bus system. It is assumed that every transmission line consists of double parallel circuits, and that a 3-L-G fault occurs at a point very close to a bus on one of parallel lines; after a while the fault is cleared by opening the faulted line.

For this condition, CUEP will be obtained based on the procedure explained in the previous section. Also the obtained CUEP is compared by carrying out the Shadowing method [12]. The obtained CUEPs provide critical values of the
transient energy of (20), which give the approximated CCTs through the common procedure of the energy function method. Note that the CCTs estimated from the energy function are only approximations providing sufficient condition for stability.

The obtained CCTs are listed in TABLES I and II for various cases for different fault locations. The tables also show the fault location and the number of iterations required for the proposed method, where \( \text{Max} \left| \frac{dx}{dt} \right| < 10^{-7} \) is used as a convergence criterion for the Newton’s method. It is observed that both the methods provide the same CCTs, implying the validity of the proposed method. It has also been confirmed that all the obtained CCTs agree with those by the conventional numerical simulation based on (19) with errors less than \( 10^{-2} \) (s).

It is noted that \( m = 2 \) is used in the proposed method for all the cases except for Case G for 3-machine system, where \( m = 18 \) is set. We should mention that case G is an unusual case, where convergence is only obtained for \( m \geq 18 \) and a larger number of iterations are required. This case was analyzed in [18], showing that the potential energy surface is not a simple shape around the exit point and CUEP, and the critical trajectory obtained was a long and winding curve.

V. CONCLUSIONS

This paper proposes a new formulation for transient stability analysis as a minimization problem for electric power systems. Different from conventional simulation methods, the formulation is not based on an initial value problem but based on boundary value problem to directly obtain a critical condition for stability such as a critical clearing time (CCT). The method is based on the computation of critical trajectory that represents a critical case for stability.

As an application of the proposed formulation, the paper presents a method for computing a controlling UEP (CUEP) based on the BCU method. The CUEP is useful for transient stability assessment by the energy function method as well as for the future extension of the proposed method.

VI. REFERENCES


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